Ecodynamic models accounting for the changes in lake ecosystem

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Abstract—Based on the basic ecological model proposed by Jørgensen and the species composition of lake ecosystem, a new approach to calculate the growth rate of phytoplankton-weighted growth rate method was presented. The thermodynamic concept “Exergy” was also introduced into the ecodynamic model for lakes, and the changes in the species composition and the ecological structure of lake ecosystem were reflected by the changing exergy. An improved ecodynamic model for lakes was developed by combining the weighted growth rate with the application of exergy. These improvements comprehensively take the ecological properties of lake ecosystems into account, making the ecodynamic model ecologically more reasonable and practically more flexible. The applications of the improved ecodynamic model in the ecological modelling and prediction of Dianchi Lake, have demonstrated that this model is practicable and satisfactory.

Keywords: weighted growth rate; exergy; ecodynamic model.

1 Introduction

More and more serious lake eutrophication has strongly affected lake’s water environmental quality and normal functions, caused a disturbance to the equilibrium of lake ecosystem, and accelerated the ageing of lake. Lake eutrophication has been a global problem and has been paid great attention to by the whole world. In essence, lake eutrophication is a process in which phytoplanktons (algae) establish a dominant growth. Therefore, the study on lake eutrophication should focus on the water quality central to phytoplankton and the relationship between phytoplankton and lake ecosystem. So far, ecological models, in which some ecological concepts and phenomena can be translated into mathematical expressions to compute and model, are very useful tools in the study of lake eutrophication. However, the parameter estimation is the “weakest point” for most of ecological models (Jørgensen, 1992a) due to either

- An insufficient number of observations to enable the modeler to calibrate the number of more or less unknown parameters;
- No, or only a little information can be found in the literature;
- Ecological parameters are in general not known with sufficient accuracy;

*: The third author.
The structure has dynamical behavior, i.e. the parameters are changing all the time or 
A combination of two or more of these points.

In this study, based on the basic ecological model proposed by Jorgensen (Jorgensen, 1976) for a shallow lake and the properties of lake ecosystem, a new approach-weighted growth rate method is presented to compute the growth rate of phytoplankton which is the crucial parameter in the ecological model. The changes in properties within the same species or even for shifting to other species with significantly different properties can be accounted for by using this approach.

In addition, the thermodynamic concept "exergy" (= biogeochemical energy of the system) (Jorgensen, 1992b) is introduced into ecodynamic model as an index to describe the changes in the ecological structure and the species composition of lake ecosystem. The changes in parameters are reflected by the changing exergy. In our previous study, exergy was used as a goal function of the maximum growth rate, settling rate and grazed rate of algae (Zhang, 1996) to study the eutrophication of Dianchi Lake. The ecological modelling of Dianchi Lake follows that the study on improvement to ecodynamic model for lakes is successful.

2 Basic ecological model

The ecological model is first proposed by Jorgensen (Jorgensen, 1976) for a lake. Fig. 1 shows the basic structure of the model.

In this model, 20 state variables involving independent cycles of carbon, nitrogen and phosphorus are considered, the differential equations and additional equations of the state variables are refereed to the literature (Jorgensen, 1976). The growth of phytoplankton is described as a two-step process: (1) uptake of nutrients following Monod's kinetics, and (2) growth determined by the internal substrate concentration. The formula is expressed as:

\[ \mu = \mu_{\text{max}} \cdot FT \cdot \left(1 - \frac{CC_{\text{min}}}{CC}\right)\left(1 - \frac{NC_{\text{min}}}{NC}\right)\left(1 - \frac{PC_{\text{min}}}{PC}\right), \]  

where \( \mu \) is the growth rate of phytoplankton, \( \mu_{\text{max}} \) is the maximum growth rate of phytoplankton, \( CC, NC \) and \( PC \) are the concentrations of carbon, nitrogen and phosphorus in algal cells, the subscript "min" denotes the corresponding minimum concentrations of \( CC, NC, PC \), and \( FT \) is the limiting factor of temperature.

![Fig. 1 Basic structure of the ecological model](image-url)
3 Improvement to ecodynamic model for lakes

3.1 Weighted growth rate

In the ecological model, the concentration of phytoplankton is modelled by the following expression:

$$\frac{d(\text{PHYT})}{dt} = (\mu - SA - GZ/Y - Q/V) \cdot \text{PHYT},$$

(2)

where \(\text{PHYT}\) is the concentration of phytoplankton, \(\mu\) and \(SA\) are the growth rate and the settling rate of phytoplankton respectively, \(GZ\) is the grazed rate of phytoplankton by zooplankton, \(Y\) is the yield of feeding zooplankton, \(Q\) and \(V\) are the outflow and the volume of the lake.

The above equation shows that the growth rate of phytoplankton. \(\mu\) is a very important parameter in the modelling of phytoplankton.

It is well known that various phytoplankton have different maximum growth rates and growth limiting factors. With the changes in external environmental factors, such as water temperature, solar radiation and input of nutrients etc., the species composition of lake ecosystem will change with time and space. Therefore, different dominant algae are given different maximum growth rates and temperature limiting factors according to their growth characteristics. After the growth rate of every dominant alga is calculated, the total growth rate of all dominant algae is weighted by multiplying every algal growth rate by its composition proportion in all dominant algae. The weighted total growth rate by this new approach is steadily changing with time and space, which accounts for the changes in lake ecosystem.

The formulae can be written as:

$$\mu(i) = \mu_{\text{max}} \cdot \text{FT}(i),$$

(3)

$$\mu_T = \sum_{i=1}^{m} \mu(i) \cdot W(i),$$

(4)

where \(\mu(i)\) is the growth rate of the \(i\)th dominant phytoplankton, \(\mu_{\text{max}}\) is maximum growth rate, \(\text{FT}(i)\) is the temperature limiting factor of the \(i\)th phytoplankton, \(W(i)\) is the composition proportion of the \(i\)th dominant phytoplankton in all phytoplanktons at considered period, \(\mu_T\) is the total growth rate weighted by all phytoplanktons, and \(m\) is the number of dominant phytoplankton.

The limiting factor of temperature \(\text{FT}\) can be calculated using the following expression:

$$\text{FT} = \begin{cases} 
0 & \text{if } T < T_{\text{min}} \\
(T - T_{\text{min}})[2(T_{\text{opt}} - T_{\text{min}}) - T] & \text{if } T_{\text{min}} \leq T < T_{\text{opt}} \\
\frac{T(2T_{\text{opt}} - T)}{T_{\text{opt}}^2} & \text{if } T \geq T_{\text{opt}}
\end{cases},$$

(5)

where \(T\) is water temperature at considered period, \(T_{\text{opt}}\) is the optimum growth rate of dominant phytoplankton, \(T_{\text{min}}\) is the minimum temperature at which phytoplankton can grow.

After the total growth rate of phytoplankton is calculated, the growth expression can be written as follows by simultaneously considering the growth limiting of nutrients (\(CC\), \(NC\) and \(PC\)): 
\[
\mu = \mu_T \cdot \left(1 - \frac{CC_{\text{min}}}{CC}\right)\left(1 - \frac{NC_{\text{min}}}{NC}\right)\left(1 - \frac{PC_{\text{min}}}{PC}\right).
\]

### 3.2 Application of exergy

According to Darwin's theory "Survival of the fittest", survival implies maintenance of biomass and growth means increase of biomass (Jorgensen, 1992c). The biomass needs to cost energy to construct biomass and then possesses exergy, which is transferable to support other exergy (energy) processes. Survival and growth can therefore be measured by use of the thermodynamic concept exergy, which may be treated as the free energy relative to the environment. Then Darwin's theory may therefore be reformulated in thermodynamic terms as follows: The prevailing conditions of an ecosystem are steadily changing and the system will continuously select the species that can contribute most to the maintenance or even growth of the exergy of the system.

Exergy is defined as:

\[Ex = T_0 \cdot (S_0 - S),\]  

where \(T_0\) is temperature and \(S_0\) entropy at thermodynamic equilibrium. From Meijer and Jorgensen (Meijer, 1979) the contribution to exergy from the nutrient cycles can be computed as follows:

\[Ex = RT \cdot \sum_{i=1}^{m} \left[C_i \ln \left(\frac{C_i}{C_{i,eq}}\right) - (C_i - C_{i,eq})\right],\]

where \(R\) is the gas constant, \(T\) is the temperature of the environment, \(C_i\) represents the concentration of the state variable of the model, \(C_{i,eq}\) is the concentration of the state variable at equilibrium, and \(m\) is the number of the state variables considered.

Based on the basic ecological model, an improved ecodynamic model is developed by combining the weighted growth rate with the application of exergy.

### 4 Modelling and prediction of Dianchi Lake ecosystem

#### 4.1 General situations of Dianchi Lake ecosystem

Dianchi Lake locates at 102°6′—47°E, 24°40′—25°02′ N, adjacent to Kunming City. It is divided into the south and north water regions by Haigeng, the north internal lake and the south external lake. It is a typical shallow lake with average depth 4.02 m, annual average water level 1886.6 m, water surface area 298.4 km² and water volume 12.0 km³.

The continual sampling at seven sampling sites for 21 months from January, 1988 to September, 1989 totally collects the phytoplanktons of 7 phyla, 16 orders, 40 families, 64 genera and 126 species, in which green alga phylum has 31 genera and 73 species, makes up 58.4 percent of the total algae, diatom phylum has 13 genera and 22 species, makes up 17.6 percent, blue alga phylum has 12 genera and 17 species, makes up 13.6 percent. These three phyla algae constitute 89.6 percent of the total algae and are the dominant algae of Dianchi Lake.

Phytoplanktons are very sensitive to their living environmental conditions, therefore, their species and distributions are markedly affected by their environmental factors, which result in the obvious difference in species composition between internal lake and external lake (Table 1). The maximum growth rates and relevant growth temperatures of the dominant algae are given in Table 2.
Table 1  The dominant algal composition of internal and external lakes

<table>
<thead>
<tr>
<th>Lake</th>
<th>Alga</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal lake</td>
<td>Green alga</td>
<td>50</td>
<td>55</td>
<td>60</td>
<td>45</td>
<td>60</td>
<td>68</td>
<td>72</td>
<td>70</td>
<td>65</td>
<td>60</td>
<td>50</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Diatom</td>
<td>27</td>
<td>30</td>
<td>23</td>
<td>30</td>
<td>25</td>
<td>16</td>
<td>10</td>
<td>13</td>
<td>15</td>
<td>20</td>
<td>36</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Blue alga</td>
<td>23</td>
<td>15</td>
<td>17</td>
<td>25</td>
<td>15</td>
<td>16</td>
<td>18</td>
<td>17</td>
<td>20</td>
<td>20</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>External lake</td>
<td>Green alga</td>
<td>30</td>
<td>38</td>
<td>35</td>
<td>25</td>
<td>20</td>
<td>30</td>
<td>28</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Diatom</td>
<td>45</td>
<td>35</td>
<td>35</td>
<td>43</td>
<td>32</td>
<td>15</td>
<td>12</td>
<td>5</td>
<td>15</td>
<td>25</td>
<td>37</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>Blue alga</td>
<td>25</td>
<td>27</td>
<td>30</td>
<td>32</td>
<td>48</td>
<td>55</td>
<td>60</td>
<td>65</td>
<td>50</td>
<td>35</td>
<td>28</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 2  The maximum growth rates and relevant growth temperatures of the dominant algae

<table>
<thead>
<tr>
<th>Alga</th>
<th>$u_{max}, 1/d$</th>
<th>$T_{max}, ^\circ C$</th>
<th>$T_{min}, ^\circ C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green alga</td>
<td>2.40</td>
<td>25</td>
<td>2</td>
</tr>
<tr>
<td>Diatom</td>
<td>0.60</td>
<td>17</td>
<td>2</td>
</tr>
<tr>
<td>Blue alga</td>
<td>0.95</td>
<td>29</td>
<td>2</td>
</tr>
</tbody>
</table>

4.2 Ecological modelling and prediction of Dianchi Lake

In Dianchi Lake ecosystem, phosphorus is the main limiting factor of nutrients, so we consider the contribution to exergy from the phosphorus cycle and adopt the following formula for computing exergy:

$$Ex_p = RT \cdot \sum_{i=1}^{m} \left[ P_i \ln \left( \frac{P_i}{P_{i, eq}} \right) - (P_i - P_{i, eq}) \right]. \quad (9)$$

In which the state variable $P_i$ includes the concentrations of phosphorus in algal cells, soluble phosphorus, phosphorus in detritus and phosphorus in zooplankton.

In practice, Dianchi Lake is divided into 5 boxes according to its hydrodynamic conditions, the inflow of the rivers, and the dividing of its water functions. Box 1 and Box 2 are in internal lake, the three other boxes are in external lake. On the basis of the dividing of the lake, the improved ecodynamic model is coupled with the box-model to develop a box-ecodynamic model for Dianchi Lake. After the analyses of sensitivity and the calibration of model parameters, the values of the main model parameters are given in Table 3. The meanings of the symbols are referred to the literature (Liu, 1991).

Table 3  The values of the main model parameters

<table>
<thead>
<tr>
<th>Box</th>
<th>$MYZ_{max}$</th>
<th>$RZ_{max}$</th>
<th>$MZ$</th>
<th>SVS</th>
<th>$KDN_{10}$</th>
<th>$KDN_{10}$</th>
<th>$UP_{max}$</th>
<th>$UN_{max}$</th>
<th>$UC_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box 1</td>
<td>0.28</td>
<td>0.015</td>
<td>0.025</td>
<td>0.15</td>
<td>0.0005</td>
<td>0.001</td>
<td>0.003</td>
<td>0.040</td>
<td>0.65</td>
</tr>
<tr>
<td>Box 2</td>
<td>0.25</td>
<td>0.020</td>
<td>0.030</td>
<td>0.17</td>
<td>0.001</td>
<td>0.005</td>
<td>0.005</td>
<td>0.025</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Note: Unit of SVS is m/24h

The differential equations of the state variables are solved by Ronge-Kutta method. Input data include the initial values of the state variables, the parameters of the box-model and the ecodynamic
model, and the daily solar radiation, water temperature and pollutant loads. In order to examine the improved ecodynamic model, we modeled the growth of phytoplankton and the main water quality norms (e.g. TN, TP) of every box in 1988, and illustrated the modeled and predicted concentrations of Box 1 and Box 3 in Fig. 2–3. Fig. 4 shows the varying process of exergy at different external phosphorus loads.

Fig. 2 The modeled results of phytoplankton
- Measured values; --- Modeled values by the improved ecodynamic model; -.- Modeled values by the unimproved ecodynamic model

Fig. 3 The predicted results of phytoplankton by the improved ecodynamic model

Fig. 4 The varying processes of exergy at different external pollutant load levels

5 Discussion

Fig. 2 follows that the improved ecodynamic model for lakes is able to model lake ecosystem more objectively than the unimproved ecodynamic model, it can model the growth of algae well in internal and external lakes. Considering the results, it seems reasonable to state that the improved
model gives a good description of the central process of lake eutrophication—the growth of algae.

5.1 Analyses of predicted results

Fig. 3 shows that, after external pollutant loads reduced by 50 percent, the growth high peaks of algae in internal lake disappear and are replaced by some low peaks, and the occurrence time of peaks is different from that of the peaks modelled at original pollutant load level. This demonstrates that the species composition in lake ecosystem will change with external pollutant loads. The occurrence of some low growth peaks of algae after external pollutant loads reduced has followed that, the release of internal pollutant loads can also provide algae with abundant nutrients although external pollutant loads have been reduced. As a result, the dredge and reduction of internal pollutant loads is an important part in the control of lake eutrophication.

In external lake, the high growth peak of algae during June-August disappears after external pollutant loads reduced by 50 percent, which means the water quality of lakes has been improved in this situation.

5.2 Analyses of exergy

It can be seen from Fig. 4, three exergy peaks occurred in internal lake during March—April, July—August and October—November respectively, while only one exergy peak appeared in external lake during June—August. The occurrence time of the exergy peak is in good agreement with that of the growth peaks of algae. From Table 1, it can be note that during the periods the exergy peaks occurred, the composition proportion of every alga in all dominant algae has obviously varied. In internal lake, the dominant algae are green alga and diatom during March—April, green alga during June—August, and green alga and diatom during October—November. In external lake, blue alga is the most dominant alga, its rapid growth in summer causes the alga bloom during June—August. It follows that the variation of species composition in lake ecosystem is often accompanied by the increase of exergy, which agrees well with Jorgensen’s results (Jorgensen, 1988).

The varying process of exergy has followed that exergy always varies with the change in external pollutant loads. Exergy decreases with the reduction of pollutant loads, but the decreasing degrees are different at various external pollutant load conditions. It is also significant that the exergy peaks gradually flat with the reduction of external pollutant loads, which results in the disappearance of the exergy peaks after the external pollutant loads reduced by 50 percent, which implies the ecological structure of lake ecosystem has varied.

The above analyses show that exergy as an index to measure lake ecosystem not only varies with the external pollutant loads, but also with the species composition and the ecological structure in lake ecosystem. The change of exergy can therefore reflect and describe the changes of water quality and lake ecosystem.

6 Conclusion

The weighted growth rate based on the species composition comprehensively takes the shifts of species in lake ecosystem and their ecological properties into account, which makes the improved ecodynamic model ecologically more reasonable and practically more flexible.
The thermodynamic concept "exergy" provides the improved ecodynamic model with an useful tool to describe lake ecosystem. The changes in species composition and ecological structure in lake ecosystem can be reflected by the changing exergy.

The improved ecodynamic model for lakes overcomes the original ecological model's defects of weak elasticity, poor predictive capacity and inadequate considerations of the ecological properties of lake ecosystem. The successful ecological modelling and prediction of Dianchi Lake have demonstrated that it is possible to develop models so as to be able to predict the changes in the properties of the species and thereby reflect possible changes in the species composition.

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(Received May 9, 1996)