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Are chaos and catastrophe theories relevant to environmental sciences?

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Abstract—Data acquired in the area of Environmental Sciences are by their very nature often discontinuous and abrupt. As such, the mathematical theories of catastrophe and chaos may be of use in analyzing such scientific data and in formulating mathematical models.

Keywords: chaos; catastrophe; environment.

INTRODUCTION

Environmental Sciences are by their nature multi-faceted. To accurately assess the extent of environmental problems, it is often imperative to quantitatively describe the parameters. Yet, the world as we know it frequently defies being reduced to a set of easily manipulable numbers.

CATASTROPHE THEORY

During the early part of last decade, catastrophe theory has emerged as a theory that can be applied to systems where there are sudden, abrupt changes, i. e. changes that are not smooth and continuous. It has been postulated that the emergence of catastrophe theory could be due to inherent limitations in classical approaches of using differential equations to describe natural phenomena which are not unusual to be discontinuous and disconnected, especially in biological systems. Relatively few phenomena are so orderly and well-behaved that they can be represented entirely by differentiable functions, as exact solutions to differential equations. The world, especially from the standpoint of an environmental scientist who needs to tackle complex problems under non-reproducible conditions, is full of unpredictable divergences and sudden transformations which call for non-differentiable functions (Fig. 1).

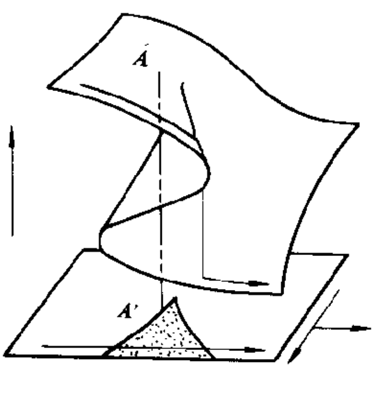

Fig. 1 A cusp catastrophe

To meet this challenge, catastrophe theory may be the answer. Table 1 depicts in mathematical form the seven elementary catastrophes which describe all possible discontinuities controlled by no more than four variables. These catastrophe models were first formulated by the French mathematician Rene Thom in 1972, which can be useful for discontinuous processes. Using Dr. Ian Stewart's simple analogy, these mathematical descriptions correspond to a mechanical system with a high degree of friction (Stewart, 1975).

Table 1 A list of the seven elementary catastrophes

Catastrophe	Energy function
Fold	$1/3x^3 + ax$
Cusp	$1/4x^4 + 1/2ax^2 + bx$
Swallowtail	$1/5x^5 + 1/3ax^3 + 1/2bx^2 + cx$
Butterfly	$1/6x^6 + 1/4ax^4 + 1/3bx^3 + 1/2cx^2 + dx$
Parabolic umbilic	$x^2y + y^4 + ax + by + cx^2 + dy^3$
Hyperbolic umbilic	$x^3 + y^3 + ax + by + cxy$
Elliptic umbilic	$x^3 - 3xy^2 + ax + by + c(x^2 + y^2)$

Catastrophe theory is a special branch of the theory of singularities, which has preoccupied mathematics for at least 300 years. One of the frequent applications of catastrophe theory is in fluid dynamics, where catastrophe theory is used to describe breaking waves as a hyperbolic umbilic catastrophe or the two dimensional inviscid fluid flow as a function of the "energy function of a catastrophe" (Poston, 1978; Zeeman, 1976; Arnold, 1984; Saunders, 1980).

It is certain that environmentalists are always concerned with the weather, which is by and large, the result of a giant, frequently turbulent, fluid system, and are concerned with gaseous and liquid discharges. It is likely that these phenomena may be modelled by one of the seven models in Table 1. On the other hand, the stability of elastic structures in engineering, such as bridges, girders and pillars, has also been analyzed by enthusiasts of catastrophe theory, e. g. Dr. G. W. Hunt discovered that a hyperbolic umbilic governs the strength of a stiffened panel (Saunders, 1980). Acoustic engineers who work on noise and vibration abatement projects may therefore find the hyperbolic umbilic model useful.

CHAOS THEORY

Just like catastrophe theory, the newly emergent chaos theory (Crutchfield, 1986; 1987; 1988; Blumel, 1990) also challenges the traditional view of modelling nature using simple deterministic systems. According to chaos theory, long term predictions of some systems are intrinsically impossible, because a system cannot be understood by breaking it down and studying it on a piecewise basis. Just as the trajectory of a flying balloon with air rushing out is not predictable, small uncertainties can lead to random behaviour even though the movements obey physical laws.

The father of chaos theory is the French mathematician Henri Poincaire who realized that unpredictability can creep into complex sysems through amplification of small fluctuations, i. e. " interaction of components on one scale can lead to complex global behaviour on a larger scale that in general cannot be deduced from knowledge of the individual components (Crutchfield, 1987; Poincoire, 1952; Hao, 1987).

To some scientists, chaos is everywhere. As an example, let us take a simple function, $y=x^2(x-1)$ for $x\leqslant 1$; $y=x-1$ for $x>1$. When one substitutes a value for x, say, $x=1.2$, one arrives at $y=0.2$. Then using this value of $y=0.2$, one can then recursively uses it as a x value, i. e. $x=0.2$, and substitute it again into the function, one arrives at $y=-0.032$. To continue repeating this process, one can design a computer program to carry out such recursive calculations indefinitely. At first sight, a random array of y values will be obtained. Yet, on close examination, one can detect bifurcation patterns or fractils. Using mathematical jargon, chaos produces fixed point-, limit cycle- or chaotic- attractors, which can be considered as geometric or topological forms that describe long-term behaviour in state space (Fig. 2).

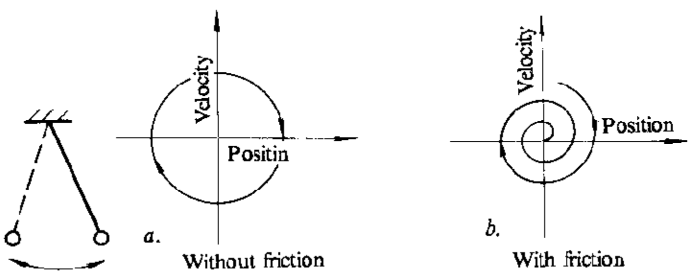

Fig. 2 A graphical representation of the "State Space"

The abstract idea of "state space" can best be visualized by the simple harmonic motion of a simple pendulum. For a simple pendulum, there are 2 degrees of freedom, viz., the pendulum's position and velocity. The pathway through which the pendulum swings is called the "state space". In a frictionless setting, the pendulum will follow a closed curve; whereas in real life situation where there is air friction and friction at the point of suspension, the pendulum's pathway will spiral inward to a point .Hence , in practical applications , and attractor can be regarded as what the behaviour of a system is attracted to , or settles down to . What appears to be random , nondirectional behaviour can have elegant , geometric structures hidden inside such behaviour, i . e . the existence of an attractorbasin portrait .

CONCLUSION

In areas where exact reproduction of identical conditions are impossible, the above mathematical considerations can be particularly useful in investigation nonlinear dynamic systems with spatially distributed degrees of freedom. Environmentalists who are concerned with non-laminar, abrupt flows, and mathematical modelling of random behaviour of dynamical systems may therefore find catastrophe and chaos theories handy to use.

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