

# A mixed integer linear programming approach for municipal solid waste management

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**Abstract**—A mixed integer linear programming (MILP) method was developed for a municipal solid waste (MSW) capacity planning problem. Based on the MILP solutions, a modelling for generating alternative (MGA) method was used for generating near-optimal alternative. Two alternatives which are different from the optimal solutions were selected and analyzed. A simulation technique was then used for incorporating stochastic parameters related to overall waste generation rate and its recyclable fraction into the modelling framework under smaller time increment conditions. It was indicated that the simulation results are useful for a more in-depth analysis of the relations between the variation of the stochastic parameters and decision schemes generated from the optimization model and between the stochastic parameters as well as for further comparison and evaluation of the generated decision alternatives. Thus, the incorporation of the simulation techniques within the optimization process provides a sound approach for more effective MSW management and planning.

**Keywords:** mixed integer linear programming; solid waste management; simulation technique.

## 1 Introduction

A significant number of municipal solid waste management (MSW) planning problems involve facility capacity issues, where a related optimization analysis will typically require the use of integer variables to indicate whether or not particular facility development or expansion options are to be undertaken. Mixed integer linear programming (MILP) is especially useful for this purpose. Kuhner and Harrington (Kuhner, 1975) studied the extension of MILP, within a partial environmental analysis framework, to solve a dynamic (multiperiod) investment model for regional solid waste management. Clayton (Clayton, 1976) relied upon MILP to investigate alternative waste systems, including the issue of individual versus regional systems. Jenkins (Jenkins, 1980) investigated the optimal location of recycling facilities for municipal solid waste within different

solid waste management systems using mixed integer linear programming. Hasit and Warner (Hasit, 1981) described WRAP (waste resource allocation program), which contains static and dynamic models solved through MILP approaches for the planning of regional solid waste systems. Jenkins (Jenkins, 1982) utilized MILP techniques, for a fixed-charge model of waste management planning in Ontario. Baetz (Baetz, 1990a) formulated a MILP model for the determination of the optimal expansion pattern for waste treatment and disposal facilities over a given time horizon. The decision variables corresponding to the development/expansion options for each facility are inherently binary, and the decision variables relating to the allocation of demand to each facility in each time period are continuous.

Two issues of concern for the MILP methods are that, firstly, the MILP model can only provide an optimal solution of decision variables, while public sector decision makers desire a range of different alternatives that can be considered when making long-term decision; secondly, in the MILP optimization program, the development and operation of the system is modelled in time increments greater than one year in an effort to make the problem computationally feasible. However, within the time increment used by the optimization program, some stochastic elements may vary substantially, which may have a significant impact upon the optimal decision schemes.

The above problems emphasize the need for further systematic approaches for the generation of near-optimal alternatives and the consideration of finer time increment. Many methods dealing with the two issues have been proposed and applied to practical problems (Hopkins, 1982; Chang, 1982; Baetz, 1990). For example, the modelling for generating alternative (MGA) methods could be used for generating near-optimal alternative solutions (Chang, 1982; Baetz, 1990b), and simulation techniques could be used for studying the system with finer time increments (Baetz, 1990a; Levin, 1983). Thus, the objectives of this paper are:

- (1) Formulate a mixed integer linear programming (MILP) model for the given MSW management planning problem, and provide optimal solutions as bases for decision making;
- (2) Apply MGA techniques for generating near-optimal alternative solution;
- (3) Integrate techniques of simulation into the optimization problem by considering finer time increments in order to reflect the effects of stochastic parameters. A comparison between the simulation results and the optimal solutions and alternatives will also be provided.

## 2 Overview of the problem

The study municipality is presently relying solely on an external landfill to satisfy waste disposal needs. The elected officials wish to develop an integrated waste management system, which will potentially require the development of a centralized composting facility, a materials recycling facility, and a waste-to-energy (W-T-E) facility (assume that a facility will either not be developed, or developed at some capacity level that will stay constant for the remainder of the planning horizon).

The centralized composting facility is for reducing organic wastes into a material suitable for

use as a soil conditioner. In a public works setting, this type of facility can often be used for wastewater sludge treatment, as well as processing of leaf/tree waste from municipal parks and households.

The materials recycling facility will help to get recyclable materials from the generation points back to a production facility, where the recyclable materials are either used in the manufacturing of the same type of product, or a range of different products. The recyclable wastes include newspaper, fine paper, glass containers, steel cans and aluminum cans.

The W-T-E facility has been used to reduce the bulk of solid waste and extract energy (by producing steam and/or electricity). This type of facility burns unprocessed waste, generally with very little front end removal of incombustible items (with the recycling program, more and more incombustible items, such as glass containers and metal cans, are being taken out at sources). However, there are concerns of air pollutant emissions and ash disposal with the W-T-E facility, which have caused significant technical and public-health concerns.

The material not handled by these facilities will be routed to an external landfill along with residue from the W-T-E facility (the W-T-E facility generate residues of approximately 30 % (on a mass basis) of the incoming waste stream). External landfill is assumed to have no capacity constraints. Fig. 1 shows the general system diagram.

Three time periods are considered, with each of them having a time interval of five years. All facilities will be operated 365 days per year, and per capita waste generation rate at year zero is 1.0 kg/(person. d). The municipality is assuming at-source waste reduction initiatives to occur over time, such that wastes requiring management at the 5-year point will be 90 % of the year zero level, and at the 10-year point will be 80 % of the year zero level. Capital expenditures on waste management facilities are limited by budget restrictions to \$  $20 \times 10^6$  (year zero \$) in any period.

It is demonstrated that the MSW generation rates for this problem vary between different time periods, and the costs of operation and transportation also vary between different facilities and different time periods (Table 1 and Table 2). Therefore, the problem under consideration is how to effectively determine which facilities should be developed, when they should be developed and at what capacity in order to achieve minimum system costs.

A mixed integer linear programming (MILP) method is considered to be a feasible approach for dealing with this problem and achieving optimal solutions.

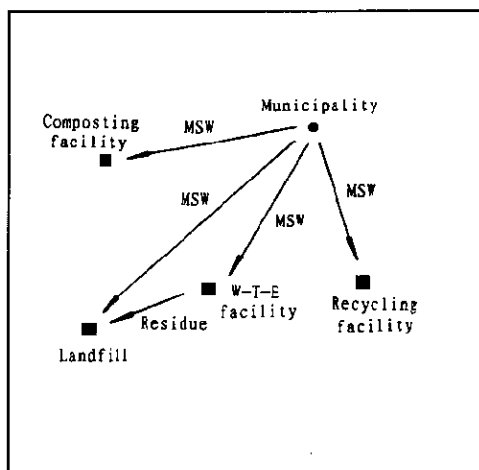


Fig.1 General system diagram

**Table 1 Facility development options and their capital costs**

	Time period		
	1	2	3
Facility development for W-T-E facility (t/d):			
$\Delta T_{11k}$	75	75	75
$\Delta T_{12k}$	150	150	150
Facility development for composting facility (t/d): (25 % of the generated wastes are compostable)			
$\Delta T_{21k}$	50	50	50
$\Delta T_{22k}$	100	100	100
Facility development for recycling facility (t/d): (40 % of the generated wastes are compostable)			
$\Delta T_{31k}$	100	100	100
$\Delta T_{32k}$	200	200	200
Capital costs of W-T-E facility development ( $10^6$ \$ present value):			
$FC_{11k}$	10	10	10
$FC_{12k}$	19	19	19
Capital costs of composting facility development ( $10^6$ \$ present value):			
$FC_{21k}$	2	1.75	1.5
$FC_{22k}$	5	4.4	3.75
Capital costs of recycling facility development ( $10^6$ \$ present value):			
$FC_{31k}$	8	7	6
$FC_{32k}$	15	13.1	11.3

**Table 2 Waste generation rates, transportation and operation costs**

	Time period		
	1	2	3
Waste generation rate (t/d):			
$WG_k$	500	450	400
Costs of transportation and operation for the W-T-E facility (\$/t):			
$C_{1k}$	50	50	50
Costs of transportation and operation for the composting facility (\$/t):			
$C_{2k}$	50	45	40
Costs of transportation and operation for the recycling facility (\$/t):			
$C_{3k}$	45	40	35
Costs of transportation and operation for the landfill (\$/t):			
$C_{4k}$	60	90	120

### 3 Systems optimization model by an MILP approach

An MILP model can be given in the following standard format:

$$\text{Min } f = C^T X, \quad (1)$$

$$\text{subject to: } AX \leq B, \quad (2)$$

$$x_j = \text{binary variable, } x_j \in X, \quad j = 1, 2, \dots, p \quad (p < n), \quad (3)$$

$$x_j = \text{continuous variable, } x_j \in X, \quad j = p + 1, p + 2, \dots, n \quad (4)$$

$$x_j \geq 0, \quad j = 1, 2, \dots, n, \quad (5)$$

where:

$$C^T = [c_1, c_2, \dots, c_n],$$

$$X^T = [x_1, x_2, \dots, x_n],$$

$$B^T = [b_1, b_2, \dots, b_m],$$

$$A = \{a_{ij}\}, \quad \forall i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n.$$

The study municipality is considered as a general system. The decision variables in the MSW management system include two categories: continuous and binary variables. The continuous variables represent the flows of the MSW to the treatment and disposal facilities over the time horizon; and the binary solutions represent the treatment facility development decisions. The objective is to achieve the minimum system cost, and the relevant flow allocation and facility development schemes. The constraints include all of the relations between the decision variables and the waste generation-treatment restrictions. The detailed MILP model for the given problem is given as follows:

$$\begin{aligned} \text{Min } f = & \sum_{i=1}^3 \sum_{m=1}^2 \sum_{k=1}^3 FC_{imk} Z_{imk} + \\ & + 1825 \left( \sum_{i=1}^4 \sum_{k=1}^3 C_{ik} X_{ik} + \sum_{k=1}^3 0.3 C_{4k} X_{1k} \right), \end{aligned} \quad (6)$$

subject to:

$$\sum_{i=1}^3 \sum_{m=1}^2 FC_{imk} Z_{imk} \leq 20 \times 10^6 \quad \forall k, \quad (7)$$

[capital expenditure constraints]

$$\sum_{i=1}^4 X_{ik} = WG_k \quad \forall k, \quad (8)$$

[waste treatment/disposal demand constraints]

$$X_{ik'} \leq \sum_{m=1}^2 \sum_{k=1}^{k'} \Delta TC_{imk} Z_{imk}, \quad i = 1, 2, 3; \quad k' = 1, 2, 3, \quad (9)$$

[facility capacity constraints]

$$X_{2k} \leq 0.25 WG_k \quad \forall k, \quad (10)$$

$$X_{3k} \leq 0.40 WG_k \quad \forall k, \quad (11)$$

[recyclable and compostable fractions constraints]

$$X_{ik} \geq 0 \quad \forall i, k, \quad (12)$$

[non-negativity constraints]

$$Z_{imk} = \begin{cases} \leq 1 \\ \geq 0 \end{cases} \quad i = 1, 2, 3; \quad \forall m, k \text{ and integer}, \quad (13)$$

[non-negativity and binary constraints]

$$\sum_{k=1}^3 Z_{imk} \leq 1 \quad i = 1, 2, 3; \quad \forall m, \quad (14)$$

[each facility development may only be considered once]

where:  $C_{ik}$  is costs of waste transportation and operation for facility  $i$  in period  $k$ , where  $i = 1$  for waste-to-energy facility,  $i = 2$  for composting facility,  $i = 3$  for recycling facility, and  $i = 4$  for landfill facility;  $k = 1, 2, 3$  for time periods 1, 2 and 3, respectively;  $FC_{imk}$  is capital costs for development capacity  $m$  at facility  $i$  in period  $k$  (\$), where  $m = 1$  for development option 1, and  $m = 2$  for development option 2;  $i = 1, 2, 3$ ;  $k = 1, 2, 3$ ;  $\Delta TC_{imk}$  is development capacity  $m$  for facility  $i$  at the start of time period  $k$  (t/d),  $m = 1, 2$ ;  $k = 1, 2, 3$ ;  $i = 1, 2, 3$ ;  $WG_k$  is average waste generation rate during time period  $k$  (t/d),  $k = 1, 2, 3$ ;  $X_{ik}$  is waste flow to facility  $i$  during time period  $k$  (t/d),  $i = 1, 2, 3, 4$ ;  $k = 1, 2, 3$ ;  $Z_{imk}$  is binary decision variable for development option  $m$  for facility  $i$  in time period  $k$ ,  $i = 1, 2, 3, 4$ ;  $m = 1, 2$ ;  $k = 1, 2, 3$ .

## 4 Method for generating near-optimal alternative solutions

In MSW management, since the real problem usually cannot be fully represented by a mathematical model, the "optimal" solution of the mathematical programming model is unlikely to be the "best" solution for the planning problem (Deininger, 1973; Roy, 1976; Baetz, 1990b; Hopkins, 1977). Even in a multiobjective framework, the best solution to the planning problem most likely lies in the inferior region of the feasible objective space defined by the model if there is at least one important unmodeled objective (Brill, 1979). Therefore, decision makers would desire a range of different alternatives that can be considered when making long-term decisions.

Furthermore, many mathematical models of MSW management systems have a large number of solutions that are nearly as good as the mathematical optimum, and some of these solutions may be better than the mathematical optimum when unmodeled issues are also taken into account. Since there are typically many good solutions, it is not practical for analysts to examine all of them directly. Therefore, modeling to generate alternative (MGA) approaches can be used to select a subset of solutions that are good with respect to modeled objectives but different from each other with respect to the values of the decision variables, such as facility location, technology, and capacity development/expansion and utilization (Baetz, 1990b; Hopkins, 1982). Analysts and decision makers can then examine these different alternatives and internalize the trade-offs between the differences in the objective function value and the differing system characteristics.

Various MGA methods have been developed and applied to practical decision making problems. For example, Brill (Brill, 1979) suggested a mathematical programming method to generate alternative solutions. This method, called Hop, Skip, and Jump (HSJ), is designed to generate alternatives that are good with respect to objectives included in the model and are significantly different from one another with respect to the decision specified. Hopkins (Hopkins, 1977) used a random generation method, which generates solutions for planning problems in stochastic simulation and statistical analysis, to generate alternative land use plans. Another

approach to generate good and different alternatives is called branch and bound/screen (BBS) method. It first generates many solutions efficiently and then apply a screening process to select solutions that are good and different (Chang, 1982). Previous applications of the MGA methods have been in the areas of water resources, land use planning, wastewater treatment, agricultural economics, and MSW management (Hopkins, 1977; 1978; Gidley, 1986; Baetz, 1990b).

In this paper, the BBS method is used to generate alternative solutions since it is more applicable to the MILP problem (Brill, 1979; Chang, 1982). The method can be used to obtain feasible solutions within a certain limit of the objective function value (specified before execution). By setting this limit appropriately, all solutions obtained will be good with respect to the modeled objective.

## 5 Simulation modelling approach

The MILP optimization method determines the optimal development pattern and facility utilization for different facility types over the planning horizon. The development and operation of the system is modelled in time increments greater than one year in an effort to make the problem computationally feasible. This level of detail is considered to be sufficient for facility planning purposes, given the significantly greater time periods required for the actual planning, design, and construction of these types of facilities. However, within a time increment considered by the optimization program, some stochastic elements may vary substantially, which may have a significant impact upon the optimal decision schemes. In this paper, two elements are given to be stochastic. They are the overall waste generation rate and its recyclable fraction.

It would not be computationally feasible for the optimization method to consider time increments small enough to capture the fluctuation in these stochastic parameters. However, a simulation model with a given development pattern as an input could feasibly model the operation of the given system under finer time increment conditions. The simulation does not attempt to optimize the capacity development patterns but requires these as modeling inputs. It generates the stochastic levels for the overall waste generation rate and its recyclable fraction. The formulas for generating the two stochastic variables are given as follows:

### 5.1 Stochastic overall waste generation rate (SOWG)

The stochastic overall waste generation rates (SOWG) are given to be uniformly distributed with the upper and lower bound values being 80 % and 120 % of the average values in each of the three time periods. Therefore, the SOWG for a time unit  $t$  ( $t = \text{one week}$ ) in planning period  $k$  is given by (Wagner, 1975; Smith, 1983):

$$SOWG_{tk} = (WG_k)[80 + (RAN)(40)]/100, \quad (15)$$

where  $RAN$  is a pseudo-random number which is uniformly distributed in interval  $(0, 1)$ , and  $WG_k$  is average waste generation rate during time period  $k$ .

### 5.2 Stochastic recyclable fraction (SRF) of the SOWG

The stochastic recyclable fraction (SRF) of the SOWG is given to have a triangular distribution, with a mean of 40 %, an upper bound of 50 % and a lower bound of 30 %. There are

two methods for generating the SRF. For the first method, assuming that the probability distribution function (PDF) of the SRF is  $f(X)$  (Fig. 2a), we can first calculate its cumulative distribution function (CDF):

$$F(x) = Pr(X \leq x) = \int_{-\infty}^x f(X) dX, \quad (16)$$

which is the sum of all the probabilities that the SRF ( $SRF = X$ ) is equal to any value less than or equal to  $x$ ,  $0 \leq F(X) \leq 1$  (Fig. 2b). Having obtain the CDF, we can use it to generate the SRF which are consistent with the distribution. The procedure is to assign the  $F(X)$  a random number  $RAN$  which is uniformly distributed in interval  $(0, 1)$ , and then find the corresponding value of SRF in the CDF curve (Smith, 1983; Ravindran, 1987).



Fig.2 The recyclable fraction (a. PDF; b. CDF)

We can also use an analytical method to derive the mathematical expression of the CDF and thus generate the SRF values. Since the SRF has a triangular distribution, we have its specific PDF as follows:

$$f(X) = (X - 30)/100 \quad \text{if } 30 \leq X \leq 40, \quad (17)$$

$$f(X) = (50 - X)/100 \quad \text{if } 40 < X \leq 50. \quad (18)$$

Thus, the CDF at  $SRF = X$  is:

$$F(X) = \int_{30}^X f(x) dx \quad \text{if } 30 \leq X \leq 40, \quad (19)$$

$$F(X) = \int_X^{50} f(x) dx \quad \text{if } 40 < X \leq 50. \quad (20)$$

Hence, for  $30 \leq X \leq 40$ , we have:

$$F_1(X) = \int_{30}^X (x - 30)/100 dx = X^2/200 - 3X/10 + 4.5. \quad (21)$$

Letting  $F_1(X) = Y_1$ , we have:

$$X = 30 + (10)(2Y_1)^{1/2} \quad \text{if } 30 \leq X \leq 40. \quad (22)$$

For  $40 < X \leq 50$ , we have:

$$F_2(X) = \int_X^{50} (50 - x)/100 dx = -X^2/200 + X/2 - 11.5. \quad (23)$$

Letting  $F_2(X) = Y_2$ , we have:

$$X = 50 - (10)(2 - 2Y_2)^{1/2} \quad \text{if } 40 < X \leq 50. \quad (24)$$

Hence, the general mathematical expressions of the CDF of the SRF is:

$$F(X) = X^2/200 - 3X/10 + 4.5 \quad \text{if } 30 \leq X \leq 40, \quad (25)$$

$$F(X) = -X^2/200 + X/2 - 11.5 \quad \text{if } 40 < X \leq 50. \quad (26)$$



Thus, when a pseudo-random number, which is uniformly distributed in interval  $(0, 1)$ , is assigned to the  $F(X)$  ( $F(X) = Y$ ), we can use Equations (22) and (24) to generate the corresponding  $SRF(SRF = X)$ . In this paper, this latter method is used.

## 6 Results

### 6.1 Result from the MILP model

Table 3 contains the optimal solutions obtained from the MILP model, where  $f$  is the objec-

Table 3 Optimal solutions obtained through the MILP model

Symbol	Decision variable, t/d			Solutions
	Facility	Capacity	Period	
$X_{11}$	W-T-E		1	0
$X_{12}$	W-T-E		2	0
$X_{13}$	W-T-E		3	0
$X_{21}$	Composting		1	100
$X_{22}$	Composting		2	100
$X_{23}$	Composting		3	100
$X_{31}$	Recycling		1	200
$X_{32}$	Recycling		2	180
$X_{33}$	Recycling		3	160
$X_{41}$	Landfill		1	200
$X_{42}$	Landfill		2	170
$X_{43}$	Landfill		3	140
$Z_{111}$	W-T-E	1	1	0
$Z_{112}$	W-T-E	1	2	0
$Z_{113}$	W-T-E	1	3	0
$Z_{121}$	W-T-E	2	1	0
$Z_{122}$	W-T-E	2	2	0
$Z_{123}$	W-T-E	2	3	0
$Z_{211}$	Composting	1	1	0
$Z_{212}$	Composting	1	2	0
$Z_{213}$	Composting	1	3	0
$Z_{221}$	Composting	2	1	1
$Z_{222}$	Composting	2	2	0
$Z_{223}$	Composting	2	3	0
$Z_{311}$	Recycling	1	1	0
$Z_{312}$	Recycling	1	2	0
$Z_{313}$	Recycling	1	3	0
$Z_{321}$	Recycling	2	1	1
$Z_{322}$	Recycling	2	2	0
$Z_{323}$	Recycling	2	3	0

System cost  $f$  ( \$ 10<sup>6</sup> )

164.9

tive function value (net system costs),  $X_{ik}$  is the waste flow to facility  $i$  during time period  $k$  ( $i = 1, 2, 3, 4$ ;  $k = 1, 2, 3$ ), and  $Z_{imk}$  is the binary decision variable for development option  $m$  for facility  $i$  in period  $k$  ( $i = 1, 2, 3$ ;  $m = 1, 2$ ;  $k = 1, 2, 3$ ).

#### 6.1.1 Facility development schemes

The results indicate that both the composting and recycling facilities should be developed at the start of period 1. The composting facility should be developed by a capacity of 100 t/d, and the recycling facility should be developed by a capacity of 200 t/d. No development is suggested for the W-T-E facility in all the three periods since it costs higher and generates residues which have to be further disposed by the external landfill.

#### 6.1.2 Waste flow allocation

The solutions for the waste flows to the composting facility indicate that the facility should be used in full load in all the three periods, i. e., the waste flows to the composting facility are determined to be 100 t/d for periods 1 to 3. The recycling facility is used to treat all recyclable wastes in all the three periods. The waste flows to the recycling facility are determined to be 200, 180, and 160 t/d for periods 1, 2 and 3, respectively. These flows are the same as the recyclable waste generation rates in the three time periods. No waste flow is planned for the W-T-E facility since it was determined not developed.

The external landfill capacity is determined only for accepting remaining waste flows which cannot be handled by the composting and recycling facilities, or exceed their upper limits of treatment capacity, because it has the highest operating and transportation costs. The waste flows to the landfill was determined to be 200, 170, and 140 t/d for periods 1, 2 and 3, respectively.

#### 6.1.3 Summary

The results indicate that the developed recycling and composting facilities are better choices of waste treatment than the external landfill facility, due to their lower operating and transportation costs. However, the constraints on facility capacities and the fraction limits of the compostable and recyclable wastes result in the solution that the waste flows cannot be completely treated by the two facilities, and the remaining parts have to be delivered to external landfill facility which is more expensive in terms of operating and transportation costs.

The composting facility has a developed capacity of 100 t/d. Therefore, the determined scheme of 100 t/d waste flows to the facility for periods 1 to 3 fully uses its available capacity although the compostable amounts of the generated waste flows in the city are 125, 112.5 and 100 t/d for periods 1, 2 and 3, respectively. The recycling facility has a developed capacity of 200 t/d. However, the determined scheme is 200, 180 and 160 t/d for periods 1, 2 and 3, respectively, because of the constraints of the recyclable waste generation rates in the city.

Since the W-T-E facility is determined not to be developed, the only option for treating/disposing the remaining waste flows is to deliver them to the external landfill facility.

Generally, for period 1, 20% of the generated waste flow should be routed to composting facility, 40% should be to the recycling facility, and another 40% to the external landfill. For period 2, 22% of the waste flow should be routed to the composting facility, 40% to the recycling facility, and 38% to the external landfill. For period 3, 25% of the waste flow should be routed to

the composting facility, 40% to the recycling facility, and 35% to the external landfill. It is indicated that 100% of the recyclable wastes are routed to the recycling facility in all the three periods, and most of compostable wastes (80%, 89% and 100% for period 1, 2 and 3, respectively) are routed to the composting facility.

## 6.2 Near-optimal alternative solutions

In this paper, BBS method is applied to generate near-optimal alternative solutions. Many feasible solutions within certain limits of the objective function value are firstly generated, and two solutions that are good and different are selected as alternatives (Table 4).

**Table 4** Near-optimal alternative solutions obtained through the MGA method

Symbol	Decision variable, t/d			Alternative 1	Alternative 2
	Facility	Capacity	Period		
$X_{11}$	W-T-E		1	0	0
$X_{12}$	W-T-E		2	0	0
$X_{13}$	W-T-E		3	0	0
$X_{21}$	Composting		1	100	100
$X_{22}$	Composting		2	38	0
$X_{23}$	Composting		3	100	100
$X_{31}$	Recycling		1	200	200
$X_{32}$	Recycling		2	180	200
$X_{33}$	Recycling		3	160	148
$X_{41}$	Landfill		1	200	200
$X_{42}$	Landfill		2	232	270
$X_{43}$	Landfill		3	140	152
$Z_{111}$	W-T-E	1	1	0	0
$Z_{112}$	W-T-E	1	2	0	0
$Z_{113}$	W-T-E	1	3	0	0
$Z_{121}$	W-T-E	2	1	0	0
$Z_{122}$	W-T-E	2	2	0	0
$Z_{123}$	W-T-E	2	3	0	0
$Z_{211}$	Composting	1	1	0	0
$Z_{212}$	Composting	1	2	0	0
$Z_{213}$	Composting	1	3	0	0
$Z_{221}$	Composting	2	1	1	1
$Z_{222}$	Composting	2	2	0	0
$Z_{223}$	Composting	2	3	0	0
$Z_{311}$	Recycling	1	1	0	0
$Z_{312}$	Recycling	1	2	0	0
$Z_{313}$	Recycling	1	3	0	0
$Z_{321}$	Recycling	2	1	1	1
$Z_{322}$	Recycling	2	2	0	0
$Z_{323}$	Recycling	2	3	0	0
System cost $f$ ( \$ 10 <sup>6</sup> )				170	175

For alternative 1, the cost limit is set at  $\$170.0 \times 10^6$  which is 3.1% higher than the cost of the optimal solution  $\$164.9 \times 10^6$ . It is indicated that the same facility development scheme as the optimal solution is obtained. However, the solutions of waste flow allocation are different. The flows to the composting facility are changed to 100, 38, and 100 t/d for periods 1, 2 and 3, respectively (in comparison with 100 t/d for periods 1 to 3 in the optimal solution), and, correspondingly, the flows to the external landfill facility are changed to 200, 232 and 140 t/d for periods 1, 2 and 3, respectively. The flows to the recycling facility are not changed. The results indicate that under this alternative, part of compostable flow (62%) originally to the composting facility in the optimal solution is shifted to the landfill facility in period 2. This provides a useful alternative for waste flow allocation when the composting facility cannot be operated at its full capacity (e.g., maintenance requirement, or other factors which make it inappropriate to run the composting facility at its full capacity in period 2).

Generally, under this alternative, 20% of the generated waste flow should be routed to composting facility, 40% should be to the recycling facility, and another 40% to the external landfill in period 1. For period 2, 8.4% of the waste flow should be routed to the composting facility, 40% to the recycling facility, and 51.6% to the external landfill. For period 3, 25% of the waste flow should be routed to the composting facility, 40% to the recycling facility, and 35% to the external landfill. It is indicated that 100% of the recyclable wastes are still routed to the recycling facility in the three periods, but less compostable wastes (34%) are routed to the composting facility and more to the external landfill in period 2.

For alternative 2, the cost limit is set at  $\$175.0 \times 10^6$  which is 6.1% higher than the cost of the optimal solution. It is indicated that the same facility development scheme as the optimal solution is still obtained. However, the solutions of waste flow allocation are different from those of the optimal solution and alternative 1. The flows to the composting facility are changed to 100, 0 and 100 t/d for periods 1, 2 and 3, respectively, the flows to the recycling facility are changed to 200, 180 and 148 t/d for periods 1, 2 and 3 respectively (in comparison with 200, 180 and 160 t/d for periods 1, 2 and 3, respectively, in the optimal solution), and, correspondingly, the flows to the external landfill facility are changed to 200, 270 and 152 t/d for periods 1, 2 and 3, respectively. The results indicate that under this alternative, no flow should be routed to the composting facility in period 2, and part of recyclable flow (7.5%) is shifted to the landfill facility in period 3. These provide useful alternatives for waste flow allocation when the composting facility is out of service or the recycling facility cannot be operated at its full capacity (e.g., maintenance requirement, or other factors which make it inappropriate to run the composting facility in period 2, or run the recycling facility in full in period 3).

Generally, under this alternative, 20% of the generated waste flow should be routed to the composting facility, 40% should be to the recycling facility, and another 40% to the external landfill in period 1. For period 2, no waste flow should be routed to the composting facility, 40% should be to the recycling facility, and 60% to the external landfill. For period 3, 25% of the waste flow should be routed to the composting facility, 37% to the recycling facility, and 38% to the external landfill. It is indicated that less compostable and recyclable wastes are routed to the

composting and recycling facilities and more to the external landfill facility compared with the optimal solution and alternative 1.

### 6.3 Simulation results and the comparison with the near-optimal alternatives

The simulation is conducted in a time increment of one week for the study time horizon. It generates the stochastic levels for the overall waste generation rates and its recyclable fraction for each of the fine time increments (weeks). Further calculation can be conducted for obtaining the generation rates of the recyclable and compostable wastes per week (compostable waste generation rate is related to the stochastic overall waste generation rate with a deterministic compostable fraction of 25 %), and the average generation rates per day. Optimal utilization of the developed facilities and the specific conditions can then be analyzed.

The simulation outputs provide useful information of the compostable, recyclable, and overall waste generation rates at each fine time increment. The variations of these stochastic parameters will directly affect the waste disposal/treatment demand constraints in the MILP model (constraints (8), (10) and (11)), and thus affect the decision. This means, when the disposal/treatment demand constraints in the MILP model become stochastic parameters, they have probabilities of oversatisfying or violating the original solution. For example, according to the simulation results, the recyclable waste generation rate could be higher than 1500 t/week with a probability of 30.8 %, or lower than 1200 t/week with a probability of 15.4 %. Thus, it is suggested that, when the uncertainty exists, it is valuable to carefully analyze both the optimization and simulation solutions in order to effectively utilize the waste treatment/disposal facilities.

The effects of different stochastic variation levels of the overall waste generation rates on the stochastic recyclable fraction and thus recyclable waste generation rates are investigated through the simulation results. Table 5 shows the relation between the stochastic overall waste generation rate and its stochastic recyclable fraction. It is indicated that there exists a trend that the stochastic recyclable fractions decrease as the overall stochastic waste generation rates increase although the effect is slight. The average recyclable fractions are 40.25 %, 40.07 % and 39.86 % corresponding to low, middle and high overall waste generation rates, respectively. The high, mid- and low levels of the stochastic overall waste generation rates represent the levels of 110 %—120 %, 95 %—105 %, and 80 %—90 % of the average waste generation rate, respectively.

**Table 5** Relation between the stochastic overall waste generation rate and the recyclable fraction

Stochastic overall waste generation rate (SOWG)	Average recyclable fraction, %
High level	39.86
Mid level	40.07
Low level	40.25

Comparing the simulation results with the three alternatives generated in the former optimization process, it is indicated that the simulation results are helpful for further analyzing and evaluating these different alternatives. For example, when it is judged, given a certain creditability criterion, that the compostable waste generation rate has a high probability of low level in period 2,

it may be more suitable to adopt alternative 1 since this alternative is determined to use less capacity of the composting facility in the period compared with the optimal solution; and when it is judged that the recyclable waste generation rate has a high probability of low level in period 3, it may be more suitable to adopt alternative 2 since this alternative is determined to use less capacity of the recycling facility in the period compared with the optimal solution and alternative 1.

The above analyses indicate that the stochastic variations of the overall waste generation rates and their recyclable fraction will influence the costs and operating policies of a developed waste management system. The degree of influence will depend on the demand characteristics and the proposed facility development and operation schemes. It is encouraged to obtain reasonable estimates of the variables so that simulation modelling results can adequately reflect the actual conditions.

## 7 Discussion

The MILP method has been found to be a feasible solution approach for the capacity planning problem that was addressed. Simulation study of the operation of the developed optimal system, incorporating the stochastic overall waste generation rate and its recyclable fraction conditions within its framework by considering finer time increments, allows a more in-depth analysis of the effects of the stochastic parameters on the MILP modeling behavior and evaluation of the generated alternatives from the optimization processes.

The major problem existing in this optimization/simulation approach is that it may not be effective when a large element of the input information is uncertain, especially when a number of coefficients of the objective function and constraints in the MILP model are uncertain. Under this situation, it may not be applicable to use the simulation technique due to the problem of computational feasibility. Although further post-optimization sensitivity analyses can be conducted, there are infinite possibilities from the uncertain information, and every sensitivity analysis run represents only the response to one or several input parameter changes. Therefore, it is suggested that grey integer programming (GIP) be introduced to solve the problem based on the concept of grey systems theory (Huang, 1992a; 1992b). In a GIP model, elements of uncertainty can be incorporated within the optimization processes and solutions through the use of grey numbers and the concepts of topological space and state. Uncertainties in the model stipulations and coefficients can be directly included in the model and communicated into the optimization processes, and thereby solutions reflecting the inherent uncertainties can be generated. Therefore, for municipal solid waste decision-making under uncertainty, the GIP model will provide an improved planning tool.

## 8 Concluding remarks

A mixed integer linear programming (MILP) method has been developed for the given MSW capacity planning problem. The MGA methods for generating near-optimal alternatives based on

the MILP model are discussed, and two alternatives which are different from the optimal solutions are selected and analyzed. A simulation technique is presented for incorporating stochastic parameters relating to overall waste generation rate and its recyclable fraction into the modelling framework under smaller time increment conditions. It is indicated that the simulation results are useful for a more in-depth analysis of the relations between the variation of the stochastic parameters and decision schemes generated from the optimization model and between the stochastic parameters, as well as for further comparison and evaluation of the generated decision alternatives. Thus, the incorporation of the simulation techniques within the optimization process provides a good extension and improvement upon the ordinary MILP method.

Problems existing in the methods are discussed, and relevant improving measures are recommended.

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