Journal of Environmental Sciences Vol.10, No.2, pp.231—237, 1998 ISSN 1001—0742

Game theory approach to optimal capital cost allocation in pollution control

Chen Wen-ying

Institute of Nuclear Energy Technology, Tsinghua University, Beijing 100084, China

Hou Dun

Urban Water Resources Center, Ministry of Construction, Beijing 100007, China

Abstract—This paper tries to integrate game theory, a very useful tool to resolve conflict phenomena, with optimal capital cost allocation issue in total emission control. First the necessity of allocating optimal capital costs fairly and reasonably among polluters in total emission control was analyzed. Then the possibility of applying game theory to the issue of the optimal capital cost allocation was expounded. Next the cooperative N-person game model of the optimal capital cost allocation and its solution ways including method based on Shapley value, least core method, weak least core methods, proportional least core method, CGA method, MCRS method and so on were delineated. Finally through application of these methods it was concluded that to apply game theory in the optimal capital cost allocation issue is helpful to implement the total emission control planning schemes successfully, to control pollution effectively, and to ensure sustainable development.

Keywords: total emission control; optimal capital cost allocation; game theory; cooperative N-person game model.

1 Introduction

Sustainable development, the new development strategy, has been paid much attention to by peoples of the world. To carry out total emission control, the effective pollution control countermeasure, is a fundamental path to fulfill sustainable development. Therefore in recent years, total emission control is greatly initiated and popularized in China.

Since in total emission control each polluter's emission reduction is obtained through optimization based on minimum cost principle, inevitably the polluters with high treatment efficiency and low marginal treatment costs shall share more emission reduction responsibility and bear higher cost burden. Of course it will frustrate enterprises' enthusiasm to treat pollution. Therefore it is important not to neglect fairness among polluters while laying stress on efficiency. One path to integrate efficiency with fairness is to allocate the total optimal capital costs among polluters fairly and reasonably in accordance with their effects on environment, so some enterprises are compensated while others pay more costs than their own optimal capital costs. This paper tries to apply game theory to study the issue.

2 Possibility of applying game theory to solve optimal capital cost allocation problem

Game theory is aimed at handling various conflict issues. Game can be divided into two-person game and N-person game based on the number of players. In 1944, publication of the famous book "Theory of games and economic behavior", written by Von Neumann and Morgenstern marked the real formation of game theory. From then on, through researches of Nash, Luce *et al*., game theory has developed continuously. Thomas pointed out that the prospects for game and game theory are inspiring, and game theory always collects the most useful technologies to analyze conflict phenomena although it can not bring satisfactory answers to all conflict problems.

When environmental quality standards are realized through cooperation based on minimum cost principle the total capital costs of all polluters, that is total optimal capital costs, is less than the sum of all polluters' deserved capital costs which are obtained on condition that each polluter threats independently its own deserved emission reduction determined in light of its impacts on environments. So how to allocate the total optimal capital costs among polluters is the same as how to allocate the cost savings resulted from cooperation(total deserved capital costs minus total optimal capital costs). Naturally, no polluter would not hope that it would share benefits as high as possible and bear costs as low as possible. Consequently conflicts exist among polluters with regard to allocating the cooperative benefits or the total optimal capital costs.

The essence of the optimal capital cost allocation issue makes it possible to be solved by applying game theory. To integrate cooperative N-person game theory with cost allocation issues is a new trend in cost allocation method studies.

3 Model formulation

Clearly, if any polluter's allocated capital costs is larger than its deserved capital costs, it would not accept such allocation result. So a reasonable cost allocation scheme must satisfy:

$$O < Z_k \leqslant X_k,\ \forall k \in K, \tag{1}$$

where, Z_k, X_k are allocated capital costs and deserved capital costs of polluter k, respectively. K donates the group formed by all polluters and is called grand coalition.

This formula is known as individual rationality condition.

In order to reduce calculation load, we only allocate total optimal capital costs among polluters instead of sources, and take the sum of deserved capital costs of the sources belonged to the same polluter as this polluter's deserved capital costs. Each polluter's optimal capital costs is calculated in the same way. So we get X_k as:

$$X_k = \sum_{i \in IK} F_i(Q_{0i} - Q_i),\ \forall k \in K, \tag{2}$$

where, $F_i(\quad)$ is pollution treatment cost function for source i; Q_{oi} is initial emission of source i; Q_i is emission permit of source i which is determined according to the environmental impact of source i; IK is a set formed by all sources belonged to polluter k.

In addition, the total allocated capital costs of all polluters must be equal to the sum of their optimal capital costs. It is formulated as:

$$\sum_{k \in K} Z_k = \sum_{k \in K} Y_k = C(K), \tag{3}$$

where Y_k is the optimal capital costs of polluter k; $C(K)$ is total optimal capital costs of all polluters.

Equation(3) is called entirety rationality condition.

When linear program is used to achieve optimization, $C(K)$ and Y_k are given by:

$$\min \sum_{i=1}^{N} F_i(\Delta Q_i),$$

$$s.t. \begin{cases} \sum_{i=1}^{N} (Q_{oi} - \Delta Q_i) f_{ij} \leqslant C_{sj}, & j = 1, 2, \cdots, M, \\ 0 \leqslant \Delta Q_i \leqslant Q_{oi}, & i = 1, 2, \cdots, N, \end{cases} \tag{4}$$

$$C(K) = \sum_{i=1}^{N} F_i(\Delta Q_i^*), \tag{5}$$

$$Y_k = \sum_{i \in IK} F_i(\Delta Q_i^*), \forall k \in K, \tag{6}$$

where $\Delta Q_i (\Delta Q_i^*)$ is emission reduction (optimal emission reduction) for source i; f_{ij} is transfer function of source i to receptor j; C_{sj} is environmental quality standard in receptor j; N is the number of polluter sources; M is receptors' number.

Besides the two conditions, there is a condition called coalition rationality condition which is mathematically state as:

$$\sum_{k \in S} Z_k \leqslant C(S), \forall S \subset K; \mid S \mid \neq 1, \tag{7}$$

where S is called coalition and is a partial group formed by some arbitrary polluters; $|S|$ is the number of polluters (players) in coalition S; $C(S)$ is the optimal capital costs of coalition S. $C(S)(S \subset K, |S| > 1$ is obtained by:

$$\min \sum_{i \in IS} F_i(\Delta Q_i),$$

$$s.t. \begin{cases} \sum_{i \in IS} (Q_{oi} - \Delta Q_i) f_{ij} \leqslant \sum_{i \in IS} Q_i f_{ij}, j = 1, 2, \cdots, M, \\ 0 \leqslant \Delta Q_i \leqslant Q_{oi}, i \in IS, \end{cases} \tag{8}$$

$$C(S) = \sum_{i \in IS} F_i(\Delta Q_i^*), \tag{9}$$

where IS is the set formed by all sources belonged to the polluters in coalition S.

The meaning of the coalition rationality condition can be easily understood thus: if the sum of the allocated capital costs of polluters in any coalition S is larger than this coalition's optimal capital costs, then these polluters would join in the coalition S instead of the grand coalition K.

In view of the above-mentioned statements, the cooperative N-person game model for the optimal capital cost allocation issue can be represented as follows:

$$\begin{cases} 0 \leqslant Z_k \leqslant X_k, \forall k \in K, \\ \sum_{k \in S} Z_k \leqslant C(S), \forall S \subset K; \mid S \mid \neq 1, \\ \sum_{k \in K} Z_K = C(K). \end{cases} \tag{10}$$

4 Model resolution

There are several ways to solve the cooperative N-person game model for optimal capital cost allocation, such as the method based on Shapley value, core method, CGA method, MCRS method and so on.

4.1 Method based on Shapley value

The optimal capital cost allocation method based on Shapley value is expressed as:

$$Z_k^* = \sum_{\substack{k \in S \\ S \subseteq K}} \frac{(\mid S \mid - 1)!(\mid K \mid - \mid S \mid)!}{\mid K \mid !}[C(S) - C(S - \{k\})], \forall k \in K, \tag{11}$$

where Z_k^* is the allocated capital costs of polluter k; $|K|$ is the number of polluters in the grand coalition K; $C(S - \{k\})$ is the optimal capital costs for coalition S with polluter k excluded.

4.2 Core method

The core of cooperative N-person game is the set of solution vectors of the game model. However, sometimes there may be no core. One of the most popular approaches to dealing with this problem is by adding a relaxing variable into the coalition rationality condition. It can be explained as: the best alternatives of some subgroups (coalitions) are very good——in a certain sense "too" good relative to the best alternative of the whole group (grand coalition), so an additional value is added to the optimal capital costs of coalition $S(1 < |S| < |K|)$ in order to encourage the whole group to stick together. No relaxing variable is added to the individual rationality condition to make sure that each polluter's allocated capital costs is not larger than its deserved capital costs.

In accordance with the different approaches of adding relaxing variable to coalition rationality condition, core method is divided into least core method, weak least core method and proportional least core method.

In least core method, uniform additional cost ε is added to the optimal capital costs of coalition $S(1<|S|<|K|)$. So the optimal capital costs allocation issue is changed to solve the following linear program:

$$\min \varepsilon$$

$$s.t. \begin{cases} 0 \leqslant Z_k \leqslant X_k, \forall k \in K, \\ \sum_{k \in S} Z_k \leqslant C(S) + \varepsilon, \forall S \subset K; \mid S \mid \neq 1, \\ \sum_{k \in K} Z_k = C(K). \end{cases} \tag{12}$$

In some cases the linear program (12) may have several solutions. If so, a device called "tie-breaking" may be used. For any imputation $Z = (Z_1, Z_2, \ldots, Z_{|K|})$ and coalition S, define the excess of S, $e(Z)$, as:

$$e(Z) = \sum_{\substack{k \in S \\ S \subset K}} Z_k - C(S). \tag{13}$$

Let $e_1(Z)$ be the largest excess of any coalition relative to X, $e_2(Z)$ the second largest excess, $e_3(Z)$ the next, and so on. The least core is the set Z_1 of all Z that minimize $e_1(Z)$. Let Z_2 be the set of all Z in Z_1 that minimize $e_2(Z)$, Z_3 the set of all Z in Z_2 that minimize $e_3(Z)$, and so on. This process eventually leads to an Z_k consisting a single imputation $\overline{Z}$, called the nucleolus. So the unique cost allocation scheme is obtained.

In weak least core method, same additional costs ε is imposed on each polluter in coalition $S(1 < |S| < |K|)$, such that:

$$\min \varepsilon$$

$$s.t. \begin{cases} 0 \leqslant Z_k \leqslant X_k, \ \forall k \in K, \\ \sum_{k \in S} Z_k \in C(S) + \varepsilon \mid S \mid, \forall S \subset K; \mid S \mid \neq 1, \\ \sum_{k \in K} Z_k = C(K). \end{cases} \tag{14}$$

The approach to resolving the nonuniqueness of linear program (14) is the "tie-breaking" device with the excess of S defined as:

$$e(Z) = \Big[\sum_{\substack{k \in S \\ S \subset K}} Z_k - C(S)\Big] / \mid S \mid. \tag{15}$$

In proportional least core method, for any coalition $S(1 < |S| < |K|)$, additional costs proportional to its optimal capital costs is imposed. The corresponding linear program becomes:

$$\min t$$

$$s.t. \begin{cases} 0 \leqslant Z_k \leqslant X_k, \ \forall k \in K, \\ \sum_{k \in S} Z_k \leqslant (1 + t) \times C(S), \forall S \subset K; \mid S \mid \neq 1, \\ \sum_{k \in K} Z_k = C(K). \end{cases} \tag{16}$$

To handle the multiple solutions of linear program(16), "tie-breaking" device is also used by defining the excess of S as:

$$e(Z) = \Big[\sum_{\substack{k \in S \\ S \subset K}} Z_k - C(S)\Big] / C(S). \tag{17}$$

4.3 CGA method

There are two key concepts in cost gap allocation (CGA) method, one is marginal cost, the other is cost gap. For the optimal capital cost allocation game, marginal cost of polluter k says that:

$$m_k^c = C(K) - C(K - \{k\}), \forall k \in K. \tag{18}$$

Marginal cost m_k^c is the lower bound that should be paid by polluter k if it wants to participate in the whole group K. That is why m_k^c is also called least allocation costs.

For each coalition S we define the cost gap of S by:

$$g^c(S) = C(S) - \sum_{k \in S} m_k^c, \forall S \subset K. \tag{19}$$

So we get the allocated capital costs of each polluter in CGA method as:

$$Z_k^* = m_k^c + \frac{\min\limits_{\substack{S \\ k \in S}} g^c(S)}{\sum\limits_{k \in K} \min\limits_{\substack{S \\ k \in S}} g^c(S)} g^c(K), \forall k \in K. \tag{20}$$

That is:

$$Z_k^* = [C(K) - C(K - \{k\})] + \frac{\min\limits_{\substack{S \\ k \in S}} \left\{C(S) - \sum\limits_{k \in S}[C(K) - C(K - \{k\})]\right\}}{\sum\limits_{k \in K} \min\limits_{\substack{S \\ k \in S}} \left\{C(S) - \sum\limits_{k \in S}[C(K) - C(K - \{k\})]\right\}} \times \left\{C(K) - \sum_{k \in K}[C(K) - C(K - \{k\})]\right\}. \forall k \in K. \tag{21}$$

4.4 MCRS method

MCRS (minimum costs-remaining savings) method is similar to CGA method. The expression to calculate each polluter's allocated capital costs is:

$$Z_k^* = Z_{k\min} + \frac{Z_{k\max} - Z_{k\min}}{\sum\limits_{k \in K}(Z_{k\max} - Z_{k\min})}\left[C(K) - \sum_{k \in K} Z_{k\min}\right], \forall k \in K. \tag{22}$$

One of the ways to obtained $Z_{k\max}$ and $Z_{k\min}$ is to solve following program:

$$\max Z_k \text{ or } \min Z_k$$

$$s.t. \begin{cases} 0 \leqslant Z_k \leqslant X_k, \forall k \in K, \\ \sum\limits_{k \in S} Z_k \leqslant C(S), \forall S \subset K; \mid S \mid \neq 1, \\ \sum\limits_{k \in K} Z_k = C(K). \end{cases} \tag{23}$$

However $Z_{k\max}$ and $Z_{k\min}(\forall k \in K)$ are only to be obtained after $2\ |K|$ linear programs with the same objectives solved. A simple way to get $Z_{k\max}$ and $Z_{k\min}$ is by taking the marginal cost and deserved capital costs of each polluter as its lower and upper bound of allocated costs respectively, that is:

$$Z_{k\min} = C(K) - C(K - \{k\}), \forall k \in K, \tag{24}$$

$$Z_{k\max} = X_k, \forall k \in K. \tag{25}$$

We call the cost allocation method based on Equation(22), (24) and (25) simplified MCRS method.

5 Application

There are four polluters with capital costs 500, 2000, 1300 and 900 thousand RMB Yuan respectively if all of them achieve their own deserved emission reduction target singly. Through solving the optimization model(4), the optimal capital costs of these polluters are obtained as 0, 2300, 900 and 600 thousand RMB Yuan respectively. These four polluters can form $10(2^4\text{-}2\text{-}4)$ kinds of coalitions. The optimal capital costs(unit is thousand RMB Yuan) of any possible coalition $S(1 < |S| < 4)$ is given in the following by the optimization model (8) and Equation(9):

$\{1,2\}=2300$ $\{1,3\}=1650$ $\{1,4\}=1250$ $\{2,3\}=2800$ $\{2,4\}=2350$
$\{3,4\}=2000$ $\{1,2,3\}=3200$ $\{1,2,4\}=2750$ $\{1,3,4\}=2450$ $\{2,3,4\}=3500$

Table 1 shows the cost allocation results with all involved linear programs solved by software GAMS.

Table 1 Optimal capital cost allocation results (Unit: 10^3RMB Yuan)

Cost allocation method	Cost allocation results
Method based on Shapley value	$Z^* = [388, 1612, 1112, 688]T$
Least core method	$Z^* = [400, 1500, 1140, 760]T$
Weak least core method	$Z^* = [412, 1463, 1163, 762]T$
Proportional least core method	$Z^* = [433, 1443, 1154, 770]T$
CGA method	$Z^* = [411, 1498, 1161, 730]T$
MCRS method	$Z^* = [387, 1524, 1159, 730]T$
Simplified MCRS method	$Z^* = [371, 1582, 1139, 708]T$

From Table 1, we can see that by using any one of above-mentioned methods every polluter's capital costs is less than its deserved capital costs(500, 2000, 1300 and 900 thousand RMB Yuan respectively) with the total capital costs minimized to 3800 thousand RMB Yuan.

6 Conclusion

Applying cooperative N-person game theory to resolve the optimal capital cost allocation issue in total emission control can not only achieve maximum total social benefits but also ensure fairness among all polluters. The above-mentioned cost allocation methods make it possible to implement the total emission control planning schemes with reasonable cost allocation results smoothly and to control pollution effectively to realize sustainable development.

References

Chen Wenying. Air pollution total emission control planning methodology and intelligent decision support system. Doctoral dissertation in Tsinghua University. 1996

Chen Bingzheng. System Engineering Theory and Practice, 1990, 10(5):30—36

Heaney J P. Water Resources Research, 1982, 18(3):476—482

Luce R D, Raiffa H. Games and decision. New York: John Willey & Sons, 1957

Thomas L C. Games theory and applications. Ellis: Horwood Limited, 1984

Tijs S N, Driessen T S H. Management Science, 1986, 32(8):1015— 1028

Von Neumann, Morgenstern O. Theory of games and economic behavior. Princeton: Princeton University Press, 1944

Young H P. Water Resources Research, 1982, 18(3):463—475

(Received for review June 11, 1997. Accepted August 18, 1997)