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Numerical simulation of stratified turbulent two-phase flow in aquatic environment

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Abstract: The two-fluid model of stratified turbulent two-phase flow in aquatic environment is developed in this paper. The motion of each phase is described by a unified multi-fluid model in an Eulerian coordinate system. The laws of turbulent transportation for each phase, and the restriction of each other between the two phases are completely simulated. The complex two-phase turbulence with strong buoyancy effects is selected to examine numerically. The extensive experimental data obtained in stratified flow are used here. Comparison of the results of numerical simulation with the experimental data is conducted. It has shown that the results of numerical simulation are satisfactory.

Key words: stratified flow; two-phase turbulence; two-fluid model; aquatic environment
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In estuary, density stratified flow exits widely because freshwater joins into seawater and the density of freshwater is different from that of seawater. It is well known that density stratification will exert a strong influence on turbulent mixing. The turbulent mixing in this kind of flow is very complicated. The two-fluid model of stratified turbulent two-phase flow is developed in this paper. The basic idea of the two-fluid model (Markatos, 1985) is that the turbulent two-phase flow is considered as the combination of the motion of each phase and the interaction of two fluids; the two turbulent fluids coexist in time and space but possess different volume fraction; the two turbulent fluids can be treated as interpenetrating continuity, which obey their own governing differential equation; the two turbulent fluids have the complex interactions in mass, momentum and energy. It could be considered that no matter how the two fluids move in a turbulent two-phase flow, the phase marks are unchangeable. Let $k=1$ and $k=2$ denote the two different fluids in the two-phase flow.

1 Mathematical model

To introduce Reynolds decomposition and average to the instantaneous Navier-Stokes equation, and to simulate the correlation terms by gradient will yield the governing equations which depict the movement of the averaged parameters of turbulent two-phase flow system.

Time-averaged continuity equation (volume fraction equation) for phase k:

$$\frac{\partial}{\partial t}(\rho_k\Phi_k) + \frac{\partial}{\partial X_j}(\rho_k\Phi_k U_{kj}) = \frac{\partial}{\partial X_j}\left[\left(\Phi_l\frac{\mu_{ek}}{\sigma_{\Phi_k}} + \Phi_k\frac{\mu_{el}}{\sigma_{\Phi_l}}\right)\frac{\partial\Phi_k}{\partial X_j}\right] + S_{k,l} + m_{k,l}. \tag{1}$$

Time-averaged momentum equation for phase k:

$$\frac{\partial}{\partial t}(\rho_k\Phi_k U_{ki}) + \frac{\partial}{\partial X_j}(\rho_k\Phi_k U_{kj}U_{ki}) = \frac{\partial}{\partial X_j}\left[\frac{\mu_{ek}}{\sigma_{\Phi_k}}\left(U_{kj}\frac{\partial\Phi_k}{\partial X_i} + U_{ki}\frac{\partial\Phi_k}{\partial X_j}\right)\right]$$
$$+ \frac{\partial}{\partial X_j}\left[\Phi_k\mu_{ek}\left(\frac{\partial U_{ki}}{\partial X_j} + \frac{\partial U_{kj}}{\partial X_i}\right)\right] - \Phi_k\frac{\partial P}{\partial X_i} + \Phi_k(\rho_k - \rho_r)g_i + F_{ki}. \tag{2}$$

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Transport equation of turbulent kinetic energy for phase k:

$$\frac{\partial}{\partial t}(\rho_k \Phi_k k_k) + \frac{\partial}{\partial X_j}(\rho_k \Phi_k U_{kj} k_k) = \frac{\partial}{\partial X_j}\left[\frac{\mu_{ek}}{\sigma_{k_k}}\Phi_k \frac{\partial k_k}{\partial X_j}\right]$$
$$+ \frac{\partial}{\partial X_j}\left[\frac{\mu_{ek}}{\sigma_{\Phi_k}} k_k \frac{\partial \Phi_k}{\partial X_j}\right] - \rho_k \Phi_k \varepsilon_k + G_k + B_k + \alpha_f \left[\frac{\mu_{ek}}{\sigma_{U_k}}\frac{\partial U_{ki}}{\partial X_i} - \frac{\mu_{el}}{\sigma_{U_l}}\frac{\partial U_{li}}{\partial X_i}\right]. \tag{3}$$

Transport equation of turbulent kinetic energy dissipation rate for phase k:

$$\frac{\partial}{\partial t}(\rho_k \Phi_k \varepsilon_k) + \frac{\partial}{\partial X_j}(\rho_k \Phi_k U_{kj} \varepsilon_k) = \frac{\partial}{\partial X_j}\left[\frac{\mu_{ek}}{\sigma_{\varepsilon_k}}\Phi_k \frac{\partial \varepsilon_k}{\partial X_j}\right]$$
$$+ \frac{\partial}{\partial X_j}\left[\frac{\mu_{ek}}{\sigma_{\Phi_k}} \varepsilon_k \frac{\partial \Phi_k}{\partial X_j}\right] + \frac{\varepsilon_k}{k_k}(C_1 G_k - C_2 \rho_k \Phi_k \varepsilon_k) - 2\alpha_f \rho \varepsilon_k, \tag{4}$$

where

$$F_{ki} = K_f \rho \Phi_k \Phi_l \,|\, U_{ki} - U_{li} \,|\, (U_{ki} - U_{li}) \varepsilon / k^{1.5}, \tag{5}$$

$$S_{k,l} = E''' = K_m \rho \Phi_k \Phi_l \,|\, U_{ki} - U_{li} \,|\, \varepsilon / k^{1.5}, \tag{6}$$

$$m_{k,l} = \left[\frac{\mu_{el}}{\sigma_{\Phi_l}} - \frac{\mu_{ek}}{\sigma_{\Phi_k}}\right]\frac{\partial \Phi_k}{\partial X_j}\frac{\partial \Phi_l}{\partial X_j}, \tag{7}$$

$$G_k = \frac{\partial U_{ki}}{\partial X_j}\left[\frac{\mu_{ek}}{\sigma_{\Phi_k}} U_{kj} \frac{\partial \Phi_k}{\partial X_i} + \mu_{ek}\Phi_k\left(\frac{\partial U_{ki}}{\partial X_j} + \frac{\partial U_{kj}}{\partial X_i}\right)\right], \tag{8}$$

$$B_k = -\frac{\mu_{ek}}{\sigma_{\Phi_k}}\frac{\rho_k - \rho_r}{\rho_k}\frac{\partial \Phi_k}{\partial X_i} g_i, \tag{9}$$

$$\alpha_f = \frac{1}{\rho} K_f E''', \tag{10}$$

$$\mu_{tk} = \rho_k C_\mu \frac{k_k^2}{\varepsilon_k}, \tag{11}$$

$$\mu_{ek} = \mu_k + \mu_{tk}, \tag{12}$$

where t is the time; the subscripts k and l are the fluid marks which may be either 1 or 2 ($k \neq l$); F_{ki} is the component of time-averaged frictional force of phase k in the i direction; U_{ki} and U_{li} are the components of time-averaged velocity of phase k and phase l in the i direction respectively; Φ_k and Φ_l are the time-averaged volume fractions of phase k and phase l respectively; ρ, k and ε are the density, turbulent kinetic energy and turbulent kinetic energy dissipation rate of mixture respectively; P is the time-averaged pressure of mixture; μ_k, μ_{tk} and μ_{ek} are the dynamic viscosity, dynamic eddy viscosity and effective dynamic viscosity of phase k respectively; ρ_k is the substance density of phase k; ρ_r is the substance density of ambient fluid; σ_{Φ_k} is the turbulent Schmidt number of phase k, which may be constant generally, but it is strongly influenced by buoyancy effects, this buoyancy influence is accounted with Munk-Anderson empirical formula (Rodi, 1980); g_i is the component of gravitational acceleration in the i direction. K_f, K_m, σ_{k_k}, σ_{ε_k}, σ_{Φ_0}, C_1, C_2 and C_μ are all empirical constants recommended in references (Rodi, 1980; Shen, 1993), giving $K_f = 0.05$, $K_m = 0.1$, $\sigma_{k_k} = 1.0$, $\sigma_{\varepsilon_k} = 1.3$, $\sigma_{\Phi_0} = 1.0$, $C_1 = 1.44$, $C_2 = 1.92$ and $C_\mu = 0.09$.

2 Method of numerical computation

The number of the differential equations of the two-fluid model is increasing up to two times as much as that of one-fluid model. And the nonlinearity and coupling of the equations become much stronger due to interactions between the two fluids. The coupling among equations is strengthened in two respects. One is the coupling among equations of the same phase, the other is that among different phases or inter-phase coupling. On the other hand the numerical solution of the equations is highly dependent upon the initial values, discretion method, iteration chart and method and relaxation ways. All these difficulties are the direct subsequence of the strong couplings. Therefore, in order to overcome these obstructions it is necessary to debase or even to cancel the nonlinearity and coupling.

It is easy to see that if the flow of two phases is described by the unified multi-fluid model, the solution could be obtained by difference method in a unified Eulerian field. Therefore, the SIMPLE procedure for solving one-phase flow can be extended to solving the two-phase flow. The pressure correction equations can be developed by using the continuity equation of mixture. Using the numerical method which approaches the two-fluid system from a no-slip one-fluid system, the numerical solutions of two-phase coupling system are carried out with under-relaxation and proper iteration chart.

3 Computation case and result discussions

3.1 Computation case

The purpose of this paper is to put on record the model's performance for strong buoyancy effects. The extensive data (Uittenbogaard, 1988) obtained in a stratified flow are used here. The experimental set-up is shown in Fig. 1. A turbulent mixing layer was created between two streams of water, initially separated by a splitter plate, having different velocities and salinity concentrations. The two streams were maintained at constant (and equal) temperature but with a density difference of 15 kg/m^3. The denser stream was below the lighter one, a combination that gives rise to stable stratification leading to suppression of turbulent mixing.

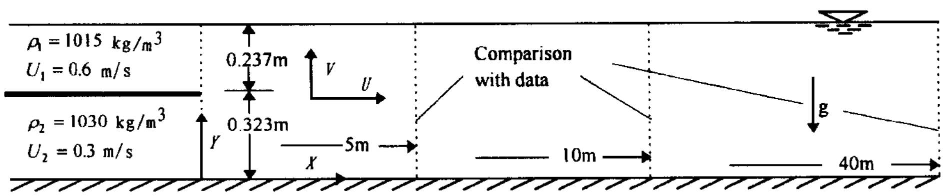

Fig. 1 Flow geometry and coordinate system

The boundary conditions at the inlet plane, comprising profiles of the dependent variables, were provided by experiment while those at exit were appropriate for fully-developed flow. The top boundary was treated as a slip plane (rigid-lid approximation) and the bottom boundary as a smooth wall where velocity follows standard log-law and turbulence is in local equilibrium.

3.2 Computational results and discussions

The predicted and measured profiles of velocity U, turbulent kinetic energy k and relative density ρ_{rel}(equal to $\frac{\rho - \rho_1}{\rho_2 - \rho_1}$) of mixture are compared in Fig. 2, 4 and 6. The predicted profiles of velocity U and turbulent kinetic energy k of two fluids have shown in Fig. 3 and 5. The profiles are presented at three streamwise locations, 5, 10 and 40 meters downstream of the splitter plate. It is immediately apparent that the predictions for velocity and relative density, while in very close agreement with the data at the early stages of development, are little different from the measurements at the last station. The predictions reproduce that the initially distinct boundary layers that were developed on either side of the spiltter plate have almost merged into a single shear layer akin to a boundary layer developing over the flume's bed. The predictions clearly reproduce that turbulent kinetic energy has completely collapsed in the outer part of the shear layer under the influence of strong stable stratification.

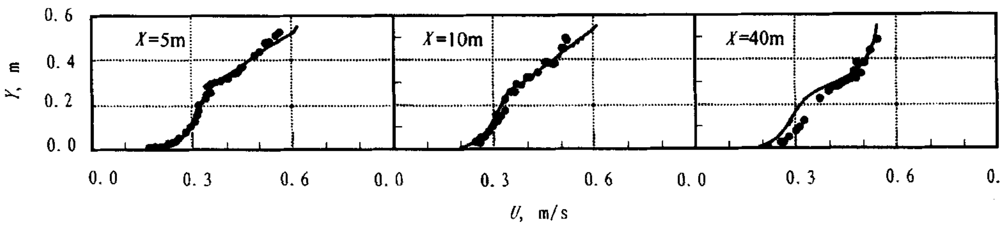

Fig.2 Predicted and measured profiles of velocity of mixture
—— predicted ● measured

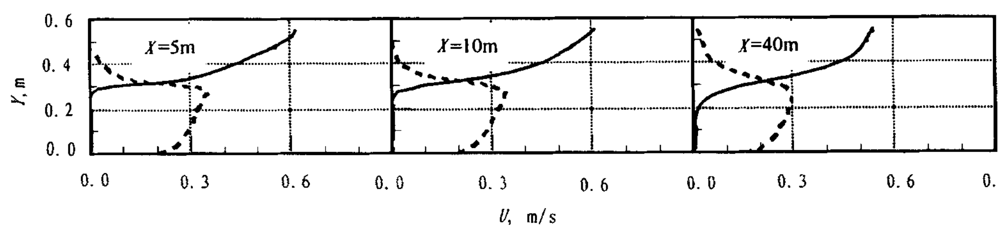

Fig.3 Predicted profiles of velocities of two fluids
—— phase one ------ phase two

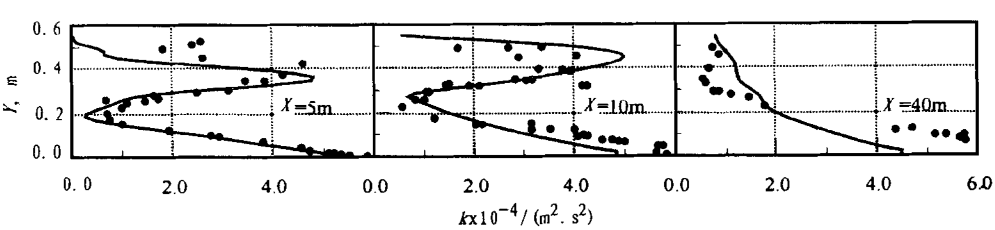

Fig.4 Predicted and measured profiles of turbulent kinetic energy of mixture
—— predicted ● measured

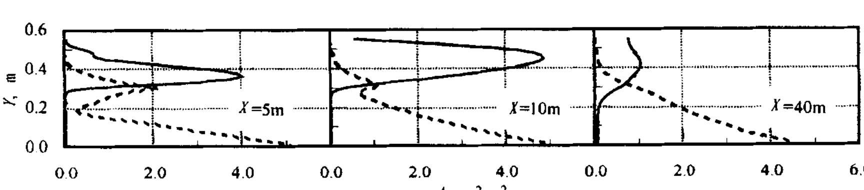

Fig.5 Predicted profiles of turbulent kinetic energy of two fluids
—— phase one ------ phase two

It can be seen from the velocity profiles and the relative density profiles that the presence of buoyancy makes the phase-coupling to be of the "Check Type", i.e. the phase-coupling does not

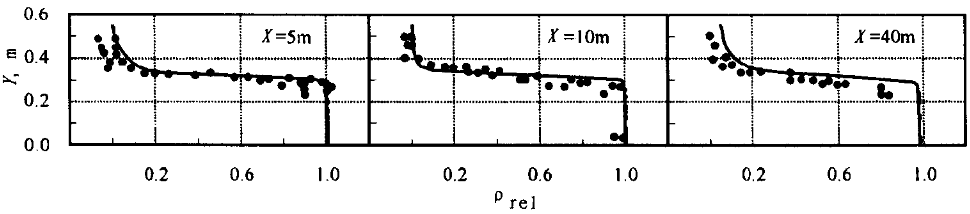

Fig.6 Predicted and measured profiles of relative density
—— predicted ● measured

make the distribution of parameters tend to be uniform but it tries to make the parameters maintain their own order. The lighter stream of upper layer flows forward and goes to diffuse into lower layer. The denser stream of lower layer flows forward and holds out against the lighter stream upwards and restrains the lighter stream from descending. The stratification of flow and density is created. Meanwhile, turbulence and entrainment between two phases are taking place. The intensity of entrainment is decreasing with the increase of density difference between two streams. It can be seen from the profiles of averaged mixture parameters that the averaged parameters are the comprehensive appearance of complex internal processes. Therefore the two-fluid model can not only simulate the "appearance" of movement of fluid but also reveal the internal processes of the complex movement of fluid, which is beyond the ability of one-fluid models.

4 Conclusions

The two-fluid model of stratified turbulent two-phase flow in aquatic environment is developed in this paper. The laws of turbulent transportation for each phase, and the restriction of each other between the two phases are completely simulated. Comparison of the results of numerical simulation with the experimental data is conducted. It has shown that the results of numerical simulation are satisfactory.

The two-fluid model of turbulent two-phase flow can not only simulate the "appearance" of movement of fluid but also reveal the internal processes of flow and mixing of two-phase flow, which is beyond the ability of one-fluid models.

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