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Two-dimensional numerical algorithm for water quality modeling in the topographically complicated river

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Abstract: In this paper, a two-dimensional numerical calculation algorithm for the water quality modeling in the Hengyang City section of the Xiangjiang River is researched considering the effect of the Dayuandu navigational key project. The research river is winding and has two branches resulted from an isle. The numerical calculation algorithm for the water quality modeling is set up on the basis of applying topographic map of the river course and the finite element method. The calculation result for the water quality modeling includes the concentration fields for various pollutants. The numerical calculation algorithm for the water quality modeling set up in this paper can be applied to shallow river with similar topographically complicated river course.

Key words: two-dimensional river; water quality; numerical calculation algorithm

1 Introduction

According to the Hunan component of the proposed inland waterways multipurpose project, the Dayuandu navigational key project, located 62 km downstream from the Hengyang City, will be built with the help of the world bank loan. After the project being constructed, the Xiangjiang River section within the Hengyang City will be changed into a reservoir resulted from the project and the water quality will be surely influenced as the river hydrological condition upstream from the project will be changed. To make it clear how much negative influence the project will have on the water quality in the Xiangjiang River section within the Hengyang City, the research river section (about 13 km) from the Tongqiaogang upstream from the Hengyang City to the Chengbei drinking water plant downstream from the Hengyang City was selected with the consideration of the pollutants' discharge distribution and the water quality management of the Hengyang City. The location of the research river section is shown in Fig. 1.

The research river section is topographically complicated as it is winding, relatively shallow with two branches resulted from an isle and its cross section changes irregularly along the river. For these reasons, the traditional one-dimensional water quality model and the traditional two-dimensional water quality model fit for straight river with uniform rectangle cross section can not be used for this research, and the two-dimensional numerical calculation algorithm based on the finite element method is proposed. To finish the water quality modeling, totally about 2921 unit cells have been divided in the researched 13 km river section and a lot of application problems have been met and solved. These problems include how to build the modeling algorithm, how to decide the width and length of the modeled unit cell, how to estimate the hydraulic and hydrological parameters required in the model and so on. As the water quality modeling problem based on the finite element method for the topographically complected river is always difficult and not well solved (Cheng, 1996; Heidtke, 1986; White, 1977), the two-dimensional numerical calculation algorithm proposed in this paper and some details used in this water quality modeling are quite possible to be used as a reference for future water quality modeling in similar topographically complicated river.

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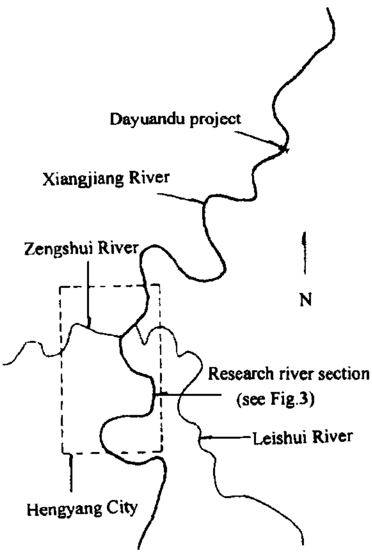

Fig.1 The location of the research river section

2 Water quality model

The model building approach for the two-dimensional numerical calculation algorithm mainly includes two steps. The first step is to divide the river flow into m flow zones following the flow direction as well as into n river sections perpendicular to the flow direction and then a series of mathematical models for all the unit cells to model the water quality based on the mass conservation law are established.

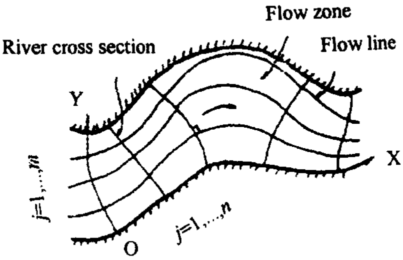

Fig.2 The rectangular curved coordinate system for the river water quality modeling

To make the research easier, the flow discharge in each flow zone has to be kept constant. In this way there will be only pollutant exchange resulting from the dispersion and not any flow discharge exchange existing between the neighboring flow zones. Correspondingly, the width for each flow zone must change along the river as the shape of the river cross section changes along the river (Fig.2).

Take one unit cell (given ij unit cell), assuming pollutant (given BOC_5) concentration is uniform in each unit cell, the pure added amount for each unit cell resulted from the river water flow is:

$$q_j(L_{i-1,j} - L_{ij}), \tag{1}$$

the pure added amount for BOD_5 in each unit cell resulted from the longitudinal dispersion is:

$$D_{(i-1,j),ij}(L_{i-1,j} - L_{ij}) - D_{ij,(i+1,j)}(L_{ij} - L_{i+1,j}), \tag{2}$$

the pure added amount for BOD_5 in each unit cell resulted from the river cross dispersion is:

$$D_{(i,j-1),ij}(L_{i,j-1} - L_{ij}) - D_{ij,(i,j+1)}(L_{ij} - L_{i,j+1}). \tag{3}$$

As the decreased BOD_5 amount in each ij unit cell is $V_{ij}K_{dij}L_{ij}$ and the input BOD_5 amount from the outside of each ij unit cell is W_{ij}^L, therefore, the following equation can be obtained according to the mass conservation principle:

$$V_{ij}\mathrm{d}L_{ij}/\mathrm{d}t = q_j(L_{i-1,j} - L_{ij}) + D'_{(i-1,j),ij}(L_{i-1,j} - L_{ij}) - D'_{ij,(i+1,j)}(L_{ij} - L_{i+1,j}) + D'_{(i,j-1),ij}(L_{i,j-1} - L_{ij}) - D'_{ij,(i,j+1)}(L_{ij} - L_{i,j+1}) - V_{ij}K_{dij}L_{ij} + W_{ij}^L, \tag{4}$$

in which

$$D'_{(i-1,j),ij} = D_{(i-1,j),ij} \cdot A_{(i-1,j),ij}/x^*_{(i-1,j),ij}, \tag{5}$$

$$D'_{ij,(i+1,j)} = D_{ij,(i+1,j)} \cdot A_{ij,(i+1,j)}/x^*_{ij,(i+1,j)}, \tag{6}$$

$$D'_{(i,j-1),ij} = D_{(i,j-1),ij} \cdot A_{(i,j-1),ij}/y^*_{(i,j-1),ij}, \tag{7}$$

$$D'_{ij,(i,j+1)} = D_{ij,(i,j+1)} \cdot A_{ij,(i,j+1)}/y^*_{ij,(i,j+1)}. \tag{8}$$

q_j is the flow in the j flow zone, m^3/s; L_{ij} the BOD_5 concentration in the ij unit cell, mg/L; V_{ij} the volume for the ij unit cell, m^3; K_{dij} the BOD_5 deoxygenating rate coefficient in the ij unit cell, 1/d; $D_{ij,kh}$ the dispersion coefficient between the ij unit cell and the kh unit cell, m^2/s; $A_{ij,kh}$ the contact surface area between the ij unit cell and the kh unit cell, m^2; $x^*_{ij,kh}$ the average longitudinal distance for the two neighboring cells ij and kh, m; $y^*_{ij,kh}$ the average crosswise distance for two neighboring cells ij and kh, m.

When the state is steady, $dL_{ij}/dt = 0$, the Equation (4) can be changed into:

$$W_{ij}^L = -q_j(L_{i-1,j} - L_{ij}) - D_{(i-1,j),ij}(L_{i-1,j} - L_{ij}) + D_{ij,(i+1,j)}(L_{ij} - L_{i+1,j}) - D_{(i,j-1),ij}(L_{ij-1} - L_{ij}) + D_{ij,(i,j+1)}(L_{i,j} - L_{i,j+1}) + V_{ij}K_{dij}L_{ij}. \quad (9)$$

Letting the BOD$_5$ concentrations in all unit cells be written in the form of $(m.n) \times l$ matrix: $L = (L_{11}L_{12}\ldots L_{1m}L_{21}L_{22}\ldots L_{ij}\ldots L_{nm})^T$, and the outside inputs for all unit cells be written in the form of $(m.n) \times l$ matrix: $W^L = (W_{11}^L W_{12}^L \ldots W_{1m}^L W_{21}^L \ldots W_{ij}^L \ldots W_{nm}^L)^T$, the matrix equation for the two-dimensional water quality modeling in the topographically complicated shallow river can be obtained as :

$$GL = W^L, \quad (10)$$

where G is the $m \times n$ BOD$_5$ matrix. According to Equation (9), each element $g_{kh}(k = 1, 2, \ldots, m; h = 1, 2, \ldots, n)$ in G can be calculated by the following equations:

$$g_{kk} = q_j + D_{(i,j-1),ij} + D_{ij,(i,j+1)} + D_{(i-1,j),ij} + D_{ij,(i+1,j)} + V_{ij}K_{dij}, \quad (11)$$

$$g_{k,k+1} = -D_{ij,(i,j+1)}, \quad (12)$$

$$g_{k,k-1} = -D_{(i,j-1),ij}, \quad (13)$$

$$g_{k,k+m} = -D_{ij,(i+1,j)}, \quad (14)$$

$$g_{k,k-m} = -q_j - D_{(i-1,j),ij}, \quad (15)$$

$$g_{k,h} = 0 \text{ (when } h \text{ is not } k+1, k-1, k+m \text{ and } k-m). \quad (16)$$

Therefore, when the outside inputs in all unit cells are known, the BOD$_5$ distribution can be obtained by:

$$L = G^{-1}W^L, \quad (17)$$

where G^{-1} is named as the responsive matrix for the two-dimensional river water quality concentration distribution.

Similar matrix equation also can be established for other pollutant with about the same approach by considering that the K_{dij} in G^{-1} will be various for different pollutant. Therefore, the concentration fields for various pollutants can be calculated when the related outside pollutant input matrix and the related responsive matrix are given.

3 Division of the flow zone

The flow discharge per unit width can be estimated by:

$$q = a(H/H^*)^b Q/B, \quad (18)$$

in which, q is the flow discharge per unit width for the river cross section, m^2/s; H the depth for one flow zone, m; H^* the average depth for the river cross section, m; Q the river flow discharge, m^3/s; B the river surface width, m.

The H, H^* and B can be decided with the help of the topographic map of the river course. The value of a and b can be decided by experience proposed by Cheng (Cheng, 1990) and Fu (Fu, 1987) given in Table 1.

Table 1 The reference value for parameters a and b

River shape	B/H^*	a, b
Straight river	$50 \leqslant B/H^* \leqslant 70$	$a = 1.0$, $b = 5/3$
Straight river	$70 \leqslant B/H^* \leqslant 100$	$a = 0.92$, $b = 7/4$
Winding river	$50 \leqslant B/H^* \leqslant 100$	$0.95 \geqslant a \geqslant 0.821$; $2.48 \geqslant b \geqslant 1.78$

After the flow discharge per unit width is decided, the flow zone number m and the width for each flow zone can be decided. Then, each unit cell's location, the length for each unit cell Δx_i, the width for each unit cell Δy_i, the depth for each unit cell can also be correspondingly decided with the topographic map of the river.

4 Treatment of the two branches resulted from an isle in the modeling river section

There is an isle locating in the modeling river section. The isle resulting in two branches when the river passes it. To treat the isle's influence on the division of the flow zones required by the modeling, firstly we use the isle inside the river to divide the river into two big flow zone (i.e. the two branches); then we use the proposed division approach for the flow zone to decide the flow zones in both of the branches to be modeled. When the flow zones are decided, one principle that should be followed is that to keep the flow discharge constant in each flow zone. The fact that the natural boundary of the isle is the best division line to satisfy this principle shows that the proposed approach to treat the branches resulted from the isle is simple and accurate.

5 The determination of the parameters used in the model

Table 2 The parameters utilized in the model

Parameter	BOD_5	COD	As	Pb
D, m^2/s	0.14	0.14	0.12	0.12
K, 1/d	0.22	0.10	0.01	0.01

The parameters in the model are decided by the measured data using the optimization method proposed by Zeng (Zeng, 1996), which are listed in Table 2.

6 The boundary condition for the calculation

6.1 The boundary condition for the pollutant discharge mouths

In the research river section from the Tongqiaogang upstream from the Hengyang City to the Chengbei drinking water plant downstream from the Hengyang City, there are 7 pollutant discharge mouths, which are Tongqiaogang, Gongnongqu, Dongzhousi, Shashimatou, Weifuchang, Bolichang, and Xiaoxiangjie pollutant discharge mouth. The pollutants discharged mainly are BOD_5, COD, As and Pb, the discharged amount of which is listed in Table 3.

Table 3 The discharged amount for each pollutant discharge mouth Unit: g/s

Pollutant	Tongqiaogang	Gongnongqu	Dongzhousi	Shashimatou	Weifuchang	Bolichang	Xiaoxiangjie
BOD_5	47.23	4.7	9.99	10.38	2.8		
COD	35.82	26.98	5.400	7.460	18.2	8.10	46.91
As	0.055	0.199	0.023	0.014	0.0028	0.0011	0.0094
Pb	0.250	0.130	0.016	0.017	0.016	0.0017	0.0031

6.2 The boundary condition for the pollutant concentrations at the initial river cross section

The initial river cross section is the Tongqiaogang section. The pollutant concentrations used as the boundary condition for the calculation are decided by the monitored values by the Hengyang Environmental Monitoring Station in 1993, which are $BOD_5 = 1.43$ mg/L, COD = 1.42 mg/L, Pb = 0.0089 mg/L and As = 0.035 mg/L.

7 Calculation procedure

(1) Applying the topographic map of the river course in 1:2000 or 1:5000 to measure the data (x_i, y_i, H_i) and estimate the average depth H_i^*; (2) calculating q_i the flow discharge per unit width for the ith river cross section with Equation (18); (3) deciding m the initial value for the flow zone and n the initial value for the river cross section; (4) calculating b_{ij} the width for jth flow zone at ith river cross section and the responsiver $x^*_{(i-1j),ij}$, $x^*_{(ij-1),ij}$, $y^*_{(i-1j),ij}$, $y^*_{(ij-1)ij}$, $A_{(i-1j),ij}$, $A_{(ij-1),ij}$ and V_{ij}; (5) calculating $D'_{(i-1j),ij}$ and $D'_{(ij-1),ij}$ with equations from (5) to (8); (6) calculating g_{kh} with equations from (11) to (16); (7) deciding W^L with initial boundary conditions; (8) calculating L with Equation (17); (9) iterating step 3 to step 8 until $|L_{ij} - L_{i-1,j}| < \varepsilon_1$ and $|L_{ij} - L_{i,j-1}| < \varepsilon_2$ to finish the calculation.

The above description is mainly about the BOD_5 modeling procedure. The calculation for other pollutants such as COD, Pb and As has about the same procedure. In the research river section (about 13 km), 2875 unit cells, each being 100m long, resulted from 126 calculation cross sections and 23 flow zones, have been used for the river water quality modeling. The ε_1 and ε_2 for BOD_5, COD, Pb and As are listed in Table 4.

8 Calculation result

Table 4 The ε_1 and ε_2 for BOD_5, COD, Pb and As

	BOD_5	COD	Pb	As
ε_1	0.0005	0.0005	0.000001	0.00001
ε_2	0.00025	0.00025	0.0000005	0.000005

The concentration fields for the BOD_5, COD, Pb and As from Tongqiaogang to the river cross section 100m downstream from the intake for the Chengbei drinking water plant (about 13 km) are calculated by applying the topographic map of the river course in 1: 2000 when the water level is 50.0m and the minimum flow discharge is 191 m^3/s. The detailed results are shown in the relative research report (The environmental impact assessment research report for Hunan inland waterway multipurpose profect submitted to Country Dept. 2, Transport Operation Div., Worldbank Washington, D. C. by Department of Environmental Engineering, Hunan University).

It is easy to find from the calculation result that there is an obvious pollution zone downstream each bigger pollution discharge mouth and that the pollution concentration decreases quickly both along the river flow direction and along the river width direction.

Compared with the second grade standard in the Surface Water Environmental Quality Standard (GB 3838-88) required in this river region, we can conclude: (1) the BOD_5 concentration exceeds the corresponding standard somewhere downstream from the Tangqiaogang pollutant discharge mouth. The length for the exceeded river region ($\geqslant$3 mg/L) is about 2000m and the corresponding area is about 0.1 km^2; (2) the COD concentration exceeds the corresponding standard ($\leqslant$4 mg/L) somewhere downstream from the pollutant discharge mouths Tongqiaogang, Gongnongqu, Dongzhousi, Weifuchang and Xiaoxiangjie. The length for the exceeded river region is about 200m and the corresponding area is about 0.01 km^2. The COD concentration in the neighboring range of all intakes for the drinking water plant does not exceed the related water quality standard; (3) the Pb concentration does not exceeds the standard ($\leqslant$0.05 mg/L) in the research river region except about 100m downstream from the Tongqiaogang pollutant discharge mouth (the corresponding area is about 0.005 km^2); (4) the As concentration anywhere in the research river region does not exceed the standard ($\leqslant$0.05 mg/L).

9 Conclusions

The two-dimensional numerical calculation algorithm for the water quality modeling in the winding topographically complicated river with two branches proposed in this paper can provide us with the water quality concentration field by utilizing the topographical map of the river course when the velocity field data can not be obtained. In this way, much work related to the velocity can be avoided but at the same time the water quality concentration field under various hydrological and hydraulic conditions still can be obtained. The proposed algorithm and the related computer program can be used for the water quality modeling in various topographically complicated river under different hydrological condition.

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