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Numerical modeling method on the movement of water flow and suspended solids in two-dimensional sedimentation tanks in the wastewater treatment plant

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Abstract: Taking the distributing calculation of velocity and concentration as an example, the paper established a series of governing equations by the vorticity-stream function method, and dispersed the equations by the finite differencing method. After figuring out the distribution field of velocity, the paper also calculated the concentration distribution in sedimentation tank by using the two-dimensional concentration transport equation. The validity and feasibility of the numerical method was verified through comparing with experimental data. Furthermore, the paper carried out a tentative exploration into the application of numerical simulation of sedimentation tanks.

Keywords: sedimentation tank; numerical simulation; vorticity; stream function

Introduction

In water treatment technology, the most commonly used method of solid liquid separation is gravitational sedimentation. From statistics, one quarter of the total investment of the wastewater treatment plant is used for sedimentation tanks (Swamee, 1996). With the development of computational hydrodynamics, the numerical method has been used to study the running states and calculate the exact value of the relevant parameters of sedimentation tanks, which are of great significances for verifying or modifying relevant numerical models and theories, presenting references for optimum design and optimizing control etc. (Chatellier, 2000).

The earlier numerical models of sedimentation tanks established by Dobbin (Dobbin, 1944) and Camp (Camp, 1940) supposed the flow states to be one-dimensional piston flows. Those models made idealized suppositions for flow state and suspended solid movement, which had limited range of applicability. With the progress of computer technology and experimental level, the emulation capacities of the numerical models of sedimentation tanks are enhanced gradually. In recent twenty years, many scholars focused the research of simulation for sedimentation tanks on two-dimensional ones for the reason that the main characteristic of the suspended solid movement in the sedimentation tank is turbulent mixing, which mostly appears in the vertical sections. For example, some scholars like Schamber and Larock (Schamber, 1981), Imam *et al*. (Imam, 1983), Adams and Rodi (Adams, 1990) all made simulation researches on the velocity field of primary sedimentation tanks by using turbulent mixing models. Zhou and McCorquodale (Zhou, 1992) calculated and analyzed the concentration of settlings in sedimentation tanks through numerical models, and also made some studies on the density currents caused by silts in secondary sedimentation tanks by using the modified k-ε closed model of turbulent flows. Because of the complexity in application of two-dimensional turbulent models, Yee-Chung Jin (Yee, 2000) did some researches on the settling process of particles with abnormal sizes in primary sedimentation tanks by using a kind of comparatively simple and applicable numerical model, of which the calculation result accorded with other models and experimental data.

In domestic, the references on the researches of numerical simulation for sedimentation tanks are seldom found. Taking the distributing calculation of velocity and concentration in typical horizontal

Foundation item: The National Natural Science Foundation of China (Nos. 70171055 and 50179011); The Teaching and Research Award Program for Outstanding Young Teachers in Higher Education Institutes in 2000; * Corresponding author rectangular sedimentation tank as an example, this paper established a series of governing equations by the vorticity-stream function method, and dispersed the equations by the control volume method, which is one of the branches in finite differencing methods. After figuring out the distribution field of velocity, the paper also calculated the concentration distribution in sedimentation tank by using the two-dimensional concentration transport equation. The validity and feasibility of the numerical method was verified through comparing with the experimental data (Imam, 1983). Moreover, the paper also carried out a tentative exploration into the application of numerical simulation of sedimentation tanks.

1 Establishment and dispersion of governing equations

1.1 Calculation objective

Generally, though the water flow in sedimentation tank is three-dimensional, it is also feasible to make two-dimensional simulation due to the reason that more considerations of the velocity influences in the vertical section on suspended solid transport will be taken into account during the simulation process. Fig. 1 shows the construction of a typical horizontal flow rectangular sedimentation tank and its idealized solution domain.

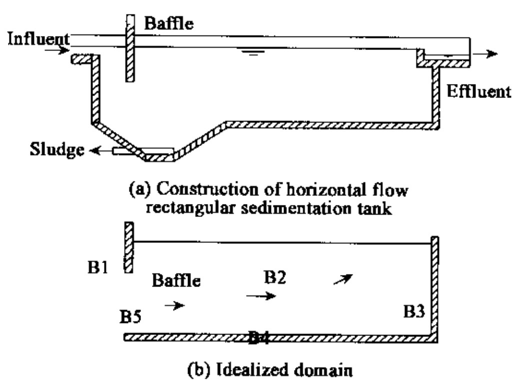

Fig. 1 Schematic diagram of horizontal flow rectangular sedimentation tank

1.2 Basic governing equations

The main characteristic of the suspended solid movement in the sedimentation tank is turbulent mixing. The basic governing equations of two-dimensional turbulent flow of an incompressible Newtonian fluid can be written as:

Reynold equation

$$\begin{cases} \frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = \overline{F_x} - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + v_{\text{eff}}\nabla^2 \bar{u} \\ \frac{\partial \bar{v}}{\partial t} + \bar{u}\frac{\partial \bar{v}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} = \overline{F_x} - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial y} + v_{\text{eff}}\nabla^2 \bar{v} \end{cases} \quad (1)$$

continuity equation $$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0, \quad (2)$$

in which $\bar{u}, \bar{v}$ = average velocities per hour; $\bar{p}$ = the average intensity pressure per hour; $\overline{F_x}, \overline{F_y}$ = x, y components of external body force per unit mass; ∇^2 = the Laplacian operator; v_{eff} = effective eddy kinematic viscosity, and $v_{\text{eff}} = v + v_t$, where v = kinematic viscosity, v_t = eddy kinematic viscosity; ρ = density; t = time.

The turbulent flow eddy viscosity μ_t passing through the solid surface will change greatly in the vertical direction of the solid surface. In the far area from the solid surface, namely thoroughly developed area, the viscosity influences will decrease rapidly, and the turbulent shearing stress will be in the highest flight, which result in $\mu_t \gg \mu$. As $v_t = \mu_t/\rho$ and $v = \mu/\rho$ the conclusion $v_{\text{eff}} \approx v_t$ could be attained in the whole idealized computing domain except for boundary areas. Different turbulent semi empirical model can be selected according to different determining method of v_t, such as eddy viscosity model, $k-\varepsilon$ two equations model etc. (Yu, 1992). v_t is supposed to be constant in this paper for convenient calculation (Imam, 1983).

1.3 Group equations of vorticity-stream function

There exist some difficulties in solving the governing equations, so through the process of vorticity and stream function import, the pressure gradient item can be removed from the momentum equation, which will simplify the solution of governing equations.

The dimensionless style can be defined as

$\tilde{x} = x/H, \tilde{y} = y/H, \tilde{u} = u/U, \tilde{t} = tU/H, \tilde{Re} = HU/v, \tilde{\omega} = \omega U/H, \tilde{\psi} = \psi/(UH)$,

in which H is the depth of the tank, and U is the horizontal average flow velocity.

The vorticity is defined as $\omega = \frac{\partial u}{\partial y} - \frac{\partial y}{\partial x}$ and the stream function is defined as $\frac{\partial \psi}{\partial y} = u, \frac{\partial \psi}{\partial x} = -v$. Take above two formulas into Reynolds Eq.(1) and continuity Eq.(2). Then the turbulent flow vorticity-stream function can be written as Formula (3) and (4) as

$$\frac{\partial \tilde{\omega}}{\partial \tilde{t}} = -\frac{\partial(\tilde{u}\tilde{\omega})}{\partial \tilde{x}} - \frac{\partial(\tilde{v}\tilde{\omega})}{\partial \tilde{y}} + \frac{1}{\tilde{Re}} \nabla^2 \tilde{\omega}, \tag{3}$$

$$\frac{\partial^2 \tilde{\psi}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{\psi}}{\partial \tilde{y}^2} = \tilde{\omega}, \tag{4}$$

in which $\tilde{R}_e$ is the characteristic Reynolds number. In order to simplify calculation, the horizontal sedimentation tank can be regarded as the forced convection neglecting body forces. So Formula (3) has ignored the influences of the body forces.

1.4 Dispersion of governing equations

Some differential methods such as the Taylor expansion method and control volume integration method etc. are applied to disperse the partial differential equations normally(Tao, 1991). Because the control volume method has a clear physical concept, from which the dispersed equations can be guaranteed to be conservational, such that it has been widely used in the flow and heat convection problems. Consequently, this paper adopts the control volume method to disperse the governing equations and the Alternative Direction Implicit (ADI) method to solve the dispersed equations(Lu, 1988).

1.4.1 Grids division

The external node method is used in this paper. The control volume and discretization method are shown in Fig.2.

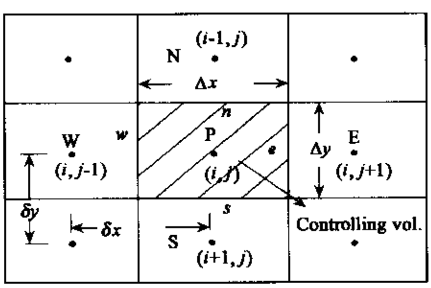

Fig.2 Control volume

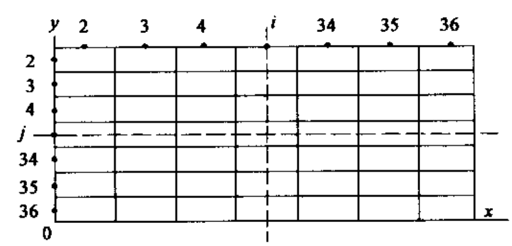

Fig.3 x, y coordinates grid system

In Fig.2, every node represents one control volume (such as the hatched place). In order to reduce the amount of calculation but not affect the accuracy of the result, the grids are comparatively thinner at the double ends of the sedimentation tank where the flow state is complicated, and thicker in the middle part of the tank. A 35 × 35 non-uniform grids system is adopted in this paper, which is shown in Fig.3 besides the x, y direction grid systems.

1.4.2 Disperse the equations

The vorticity, stream, velocity and concentration functions can all be written as the following common subscript form shows.

$$\frac{\partial(a_\phi \phi)}{\partial t} + \frac{\partial(a_\phi u_j \phi)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\Gamma_\phi \frac{\partial \phi}{\partial x_j}\right] + S_\phi. \tag{5}$$

To different variable ϕ, the values of α_ϕ, Γ_ϕ, S_ϕ in Formula (5) are listed in Table 1.

Table 1 Dispersion coefficients of governing equations

Style	Vorticity function $\frac{\partial\bar{\omega}}{\partial t} = -\frac{\partial \overline{u\omega}}{\partial x} - \frac{\partial \overline{v\omega}}{\partial y} + \frac{1}{\bar{R}e}\nabla^2\bar{\omega}$	Stream function $\frac{\partial^2\bar{\psi}}{\partial x^2} + \frac{\partial^2\bar{\psi}}{\partial y^2} = \omega$	Concentration function $\frac{\partial C}{\partial t} = -\frac{\partial uC}{\partial x} - \frac{\partial vC}{\partial y} + \bar{V}_s\frac{\partial C}{\partial y} + \frac{1}{RS_h}$	Style	Vorticity function $\frac{\partial\bar{\omega}}{\partial t} = -\frac{\partial \overline{u\omega}}{\partial x} - \frac{\partial \overline{v\omega}}{\partial y} + \frac{1}{\bar{R}e}\nabla^2\bar{\omega}$	Stream function $\frac{\partial^2\bar{\psi}}{\partial x^2} + \frac{\partial^2\bar{\psi}}{\partial y^2} = \omega$	Concentration function $\frac{\partial C}{\partial t} = -\frac{\partial uC}{\partial x} - \frac{\partial vC}{\partial y} + \bar{V}_s\frac{\partial C}{\partial y} + \frac{1}{RS_h}$
ϕ	$\bar{\omega}$	$\bar{\psi}$	C	D_e	$\Delta y/(\delta x)_e$	$\Delta y/(\delta x)_e$	$\Delta y/(\delta x)_e$
α_ϕ	1	0	1	D_w	$\Delta y/(\delta x)_w$	$\Delta y/(\delta x)_w$	$\Delta y/(\delta x)_w$
Γ_ϕ	$1/\bar{R}e$	1	$1/RS_h$	D_n	$\Delta x/(\delta y)_n$	$\Delta x/(\delta y)_n$	$\Delta x/(\delta y)_n$
S_ϕ	0	$-\bar{\omega}$	$\bar{V}_s\partial C/\partial y$	D_s	$\Delta x/(\delta y)_s$	$\Delta x/(\delta y)_s$	$\Delta x/(\delta y)_s$

Limited to the space of this paper, the detailed integral derivation process of the governing equations can be found in reference(Tao, 1991). From Fig.2, the final dispersed equation of Formula (5) can be written as

$$\alpha_p\phi_p = \alpha_E\phi_E + \alpha_W\phi_W + \alpha_N\phi_N + \alpha_S\phi_S + b, \quad (6)$$

in which the value of every parameter is defined in Table 2.

Table 2 Calculation expressions of coefficients of the differencing equations

Coefficients	Computing formulas	Coefficients	Computing formulas	Coefficients	Computing formulas
a_E	$D_eA(\lvert P_e\rvert) + \Vert -F_e, 0\Vert$	a_s	$D_sA(\lvert P_s\rvert) + \Vert -F_s, 0\Vert$	a_P	$a_E + a_W + a_N + a_S + a_P^0 - S_P\Delta x\Delta y$
a_w	$D_wA(\lvert P_w\rvert) + \Vert -F_w, 0\Vert$	a_p^0	$\frac{\rho_P^0\Delta x\Delta y}{\Delta t}$		
a_N	$D_nA(\lvert P_n\rvert) + \Vert -F_n, 0\Vert$	b	$S_c\Delta x\Delta y + a_P^0\phi_P^0$		

In Table 2, ϕ_P^0 and ρ_P^0 are given values corresponding to moment t, other values such as ϕ_P, ϕ_E, ϕ_W, ϕ_N, ϕ_S are unknown values at moment $t + \Delta t$; ρ is the fluid density and u is the flow velocity on boundary surface; S_C and S_P can be obtained by linearization process from the source item: $S_\phi = S_P + S_P\phi_P$; $\Vert$ A, B $\Vert$ represents the maximum value among A and B; to different variable ϕ, the definition formulas of D_e, D_w, D_n and D_s, which are diffusion conductance in four directions of the grid boundary, can be found in Table 1; F_e, F_w, F_n and F_s are flow masses on four boundaries; P_e, P_w, P_n and P_s are called Peclet numbers, of which the definition functions are listed in Table 3.

Table 3 Flow mass and Peclet number calculation expressions

Boundary mass flow	Computing formulas	Peclet number	Computing formulas	Boundary mass flow	Computing formulas	Peclet number	Computing formulas
F_e	$(\rho u)_e\Delta y$	P_e	$\frac{F_e}{D_e}$	F_n	$(\rho u)_n\Delta y$	P_n	$\frac{F_n}{D_n}$
F_w	$(\rho u)_w\Delta y$	P_w	$\frac{F_w}{D_w}$	F_s	$(\rho u)_s\Delta y$	P_s	$\frac{F_s}{D_s}$

The determination of function $A(\lvert p_i\rvert)$, $(i = e, w, n, s)$ has different finite difference schemes, such as the windward, mixed and exponential function schemes etc. This paper selects the exponential function schemes namely $A(\lvert p_i\rvert) = \Vert 0, (1-0.1)\lvert p_i\rvert^5\Vert$.

1.4.3 Initial and boundary conditions

To integrated incompressible hydrokinetic problems, the initial conditions are of unimportance to steady results(Li, 1989). The same velocity fields can be acquired though different initial conditions are adopted in this paper. For convenience, the symbol " ~ " of all following non-dimensional variables are removed.

With reference to the boundaries $B_1 \sim B_5$ in Fig. 1, the boundary conditions could be given as following:

B_1 and B_3 are impermeable solid surfaces, so $\psi = 0$, ω can be attained by second order Woods equation(Tao, 1991)

$$\omega_{i,1} = \frac{3(\psi_{i,2} - \psi_{i,1})}{(\Delta y)^2} - \frac{1}{2}\omega_{i,2}. \tag{7}$$

B_2 is the flow surface, so $\psi = 1, \omega = 0$; B_4 is the flow line, so $\psi = 0$ and ω can be figured out by $\omega_{i,1} = \omega^* H/U, \omega^*$ is the wall vorticity; B_5 is the inflow zone, ψ can be obtained by Formula (8):

$$\psi = \begin{cases} u_c\left\{y + 0.055\left[\left(1 - \frac{y}{0.15}\right)^{2.75} - 1\right]\right\} & 0 < y < 0.15 \\ u_c(y - 0.055) & 0.15 \leqslant y \leqslant H_B - 0.08, \\ u_c\left\{y - 0.055 - 0.027\left[\frac{y - (H_B - 0.08)}{0.08}\right]^3\right\} & (H_B - 0.08) < y < H_B \end{cases} \tag{8}$$

ω can be acquired from $\omega = \partial u/\partial y$, which can be written as:

$$w = \begin{cases} 11.667\left[\frac{0.15 - y}{0.15}\right]^{0.95} u_c & 0 < y < 0.15 \\ 0 & 0.15 \leqslant y \leqslant H_B - 0.08, \\ \frac{25}{0.08}\frac{y - (H_B - 0.08)}{0.08} u_c & (H_B - 0.08) < y < H_B \end{cases} \tag{9}$$

in which, $H_B = H = H_{\text{baffle}}$, H is the depth of sedimentation tank, and H_{baffle} is the submerged depth of the baffle; u_c is the normalized core jet velocity.

2 Model calculation and verification

The solving procedures of the model are listed as follows: (1) Give a new value $\psi_{i,j}^{(0)}$ to the stream function, from which the boundary vorticity $\omega_{i,j}^{(0)}$ can be obtained. Figure out the velocity u and v by the velocity equation. Then, the coefficients of the ω function can be determined. (2) Solve the vorticity function namely Formula (3) to acquire the vorticity value $\omega_{i,j}^{(1)}$ of every node. (3) Solve the stream function by the vorticity values to acquire the stream function value $\psi_{i,j}^{(1)}$ of every node. (4) Renew the coefficients and boundary vorticity of the ω dispersed function by the stream function values. Repeat above procedures until the convergent result is obtained. (5) Compute the flow velocities u and v. (6) Solve the concentration function by u and v to get the concentration value of every node.

In order to verify the feasibility and effectiveness of this method, this paper has compared the experimental results of Imam(Imam, 1983) with the results worked out by the numerical method introduced in this paper. The basic parameters of the sedimentation tank are listed in Table 4, and part of the comparing result is shown in Fig.4 and Fig.5.

Table 4 Basic parameters of sedimentation tank

	Flow volume (q), cm^3/(s·cm)	Depth (H), cm	Length (L), cm	Length of baffle (H_B), cm
Measurement of velocity	45.2	10.3	73	5.5
Measurement concentration	109	11.9	73	5.0

From above comparing figures, it is clear that the computing result is well accordant with the experimental result. The values of u, x and y are all normalized.

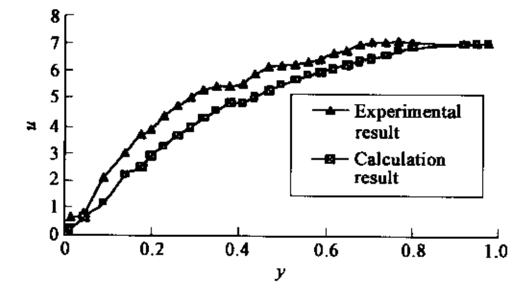

Fig.4 Comparison figure of measurement and calculation of horizontal velocity at $x = 4.85$

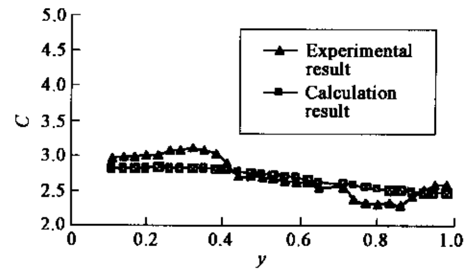

Fig. 5 Comparison figure of measurement and calculation of concentration at $x = 4.85$

3 Application of numerical simulation of sedimentation tank

In order to explore the application of numerical simulation of sedimentation tank, this paper set an example by studying the influence of submerging depth of the baffle on settling efficiency of sedimentation tank. The computing conditions is shown in Table 4, fix the values of all parameters except the length of the baffle. According to different baffle submerging depth, calculate different settling efficiencies by numerical simulation method introduced in this paper. The result is shown in Fig.6 after calculation. It is can be seen that, when the depth is changing from 0.3 – 0.7, the settling efficiency is decreasing, and when the depth is lower than 0.3, the efficiency is increasing. Therefore, 0.3 is the best submerging value.

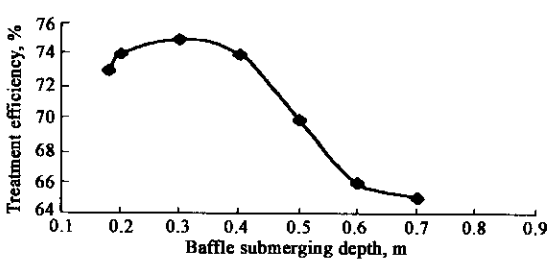

Fig.6 Relationship between baffle submerging depth and treatment efficiency

From intuitive analysis, in some extent, with the decreasing process of the submerging depth, the vorticity behind the baffle is decreasing and the jet under the baffle is abating which will reduce the eddy diffusivity. On the other hand, when the submerging depth of the baffle is increasing, the adlittoral operational characteristic will take predominance and the dead space will expand, which can decrease the settling efficiency. Furthermore, if the submerging depth is too short, it is inclinable to induce the dispersion and reflux processes, which will retroact for the settling efficiency. Therefore, how to choose a suitable submerging depth of the baffle has need of great consideration. The numerical simulation method can take an important role in this field.

4 Conclusions

The research result shows that the vorticity-stream function can be successfully applied in the numerical simulation of sedimentation tanks with the advantages of simplicity and convenience, omission of solving the pressure item, and reduction of solving variables that could save great computing time.

Using the numerical simulation method to study and analyze the designing parameters of sedimentation tanks theoretically is of great significance to guide the design and optimization of sedimentation tanks.

There are still lots of deficiencies of the vorticity-stream function method. For example, in three-dimensional cases, the stream function does not exist, so the method in this paper can not be popularized into high dimensional conditions. Moreover, some suppositions and simplicities are adopted during the calculation of the models, which would affect the result precision. But with no doubt, the numerical simulation method will be more widely used in the field of environmental engineering for its powerful vitality.

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