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Prediction of the amount of urban waste solids by applying a gray theoretical model

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Abstract: Urban waste solids are now becoming one of the most crucial environmental problems. There are several different kinds of technologies normally used for waste solids disposal, among which landfill is more favorable in China than others, especially for urban waste solids. Most of the design works up to now are based on a roughly estimation of the amount of urban waste solids without any theoretical support, which lead to a series problems. To meet the basic information requirements for the design work, the amount of the urban waste solids was predicted in this research by applying the gray theoretical model GM (1, 1) through non-linear differential equation simulation. The model parameters were estimated with the least square method (LSM) by running a certain MATALAB program, and the hypothesis test results show that the residual between the prediction value and the actual value approximately comply with the normal distribution $N(0, 0.21^2)$, and the probability of the residual within the range (−0.17, 0.19) is more than 95%, which indicate obviously that the model can be well used for the prediction of the amount of waste solids and those had been already testified by the latest two years data about the urban waste solids from Loudi City of China. With this model, the predicted amount of the waste solids produced in Loudi City in the next 30 years is 8049000 ton in total.

Keywords: gray theoretical model; prediction; waste solids

Introduction

Urban waste solids are mostly produced from residential areas, factories or manufactures, marketplaces, hotels, restaurants, streets and other functional areas. With the population increasing all over China, urban waste solids are now becoming one of the most crucial environmental problems. Three technologies are normally used for solving this problem, including landfill, incineration and composite, however, in China landfill is more favorable than others. To meet the basic information for the design work of a landfill construction, the amount of the urban waste solids has to be determined first. The factors which affect directly or indirectly the amount of urban waste solids mainly include urban population, living condition, how developed of the local business, industrialization level and so on. In China, most of the design works, at present, are based on a roughly estimation of the amount of urban waste solids without any theoretical support, which lead to a series problems. In this study, a gray theoretical model was developed for prediction of the amount of the urban waste solids, which provides a basic tool for the landfill design.

Generally, the urban waste solids system includes both known and unknown information, which make it suitable to apply a gray theory system to describe the variation of the amount of waste solids. Furthermore, the characteristics of the amount of urban waste solids: flat increase by degrees, nonnegative, uneven variety, are all consistent with the mathematical character of gray theory, which make it possible to construct a gray theoretical model. On the basis of the gray system theory, with the ready data of the amount of waste solids, which are various with different years, a gray theoretical model could be constructed from the data characterized by dynamic memory to reveal the internal rule of the variation of the amount of urban waste solids, which can be used for the prediction of the amount of waste solids.

1 The fundamental and method of the gray theory

The previous research works(Deng, 1987; Zeng, 1994) have shown that the gray theory model GM (1, 1) can be used for simulating the serials complied with the variety law of index $\{x^{(1)}(i)\}(i = 1, 2, 3, \cdots)$ with a high precision($\{x^{(1)}(i)\}$ is from the once accumulation of the original serials' $\{x^{(0)}(i)\}$). But in the case of that the serials from once accumulation does not comply with the variety law of index, twice accumulation can be made to built a new serial. In this paper a non-linear differential equation, shown as Equation (1) as following, was taken for the new serials' simulation.

$$\frac{dx^{(2)}(k)}{dk} + d'_1(1)[x^{(2)}(k)]^{d_1(3)} = d_1(2), \quad (1)$$

$$x^{(2)}|_{k=0} = x^{(2)}(1),$$

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i),$$

$$x^{(2)}(k) = \sum_{i=1}^{k} x^{(1)}(i).$$

The Equation (1) can be expressed in another way as following:

$$\frac{\Delta x^{(2)}(k)}{\Delta k} + d'_1(1)[x^{(2)}(k)]^{d(3)} = d_1(2). \quad (2)$$

Where $\Delta x^{(2)}(k)$ is the increment within the time Δk. When Δk has a given value 1, according to the gray system theory, $\Delta x^{(2)}(k)$ can be obtained from the average of the background value.

When $\Delta x^{(2)}(k)(k = 2,3,4,\cdots)$ is replaced by $\frac{1}{2}[x^{(2)}(k) + x^{(2)}(k-1)]$, following equation can be obtained from Equation(2):

$$\frac{\Delta x^{(2)}(k)}{\Delta k} = d_1(2) - d'_1(1)\left[\frac{1}{2}(x^{(2)}(k) + x^{(2)}(k-1))\right]^{d_1(3)}.$$

When Δk is 1, $\Delta x^{(2)}(k) = x^{(2)}(k) + x^{(2)}(k-1) = x^{(1)}(k)$, in which $k \geqslant 2$

Let $Y = \{x^{(1)}(2), x^{(1)}(3), \cdots, x^{(1)}(k)\}$,

$L = \frac{1}{2}[x^2(k) + x^{(2)}(k-1)], (k \geqslant 2)$.

When $-d'_1(1)$ is shown by $d_1(1)$, the Equation (2) can be shown as

$$Y = d_1(1)L^{d_1(3)} + d_1(2). \quad (3)$$

The key of the model is to determinate the parameters $d_1(1)$, $d_1(2)$ and $d_1(3)$. Combined the equations above, a model used for prediction of the amount of the waste solids was constructed shown as Equations (4). The amount of the waste solid represented by $x^{(0)}(k)$ in Equation (4) can be worked out through the recursion equation:

$$\left\{\begin{array}{l} x^{(2)}(1) = x^{(0)}(1); \\ x^{(2)}(k) - x^{(2)}(k-1) = d_1(1)\left[\frac{1}{2}(x^{(2)}(k) + x^{(2)}(k-1))\right]^{d_1(3)} + d_1(2); \\ \hat{x}^{(1)}(k) = x^{(2)}(k) - x^{(2)}(k-1); \\ \hat{x}^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1). \end{array}\right\} \quad (4)$$

2 Model parameters determination

With the recent 9 years annual amount of waste solids produced in Loudi City (Table 1), the model parameters $d_1(1)$, $d_1(2)$, and $d_1(3)$ can be figured out by the least square method (LSM) through applying a MATALAB program. Then the different value and the prediction amount of the waste solids, can be worked out through the ordinal iterative by model calculation.

Table 1 The amount of waste solids during past 9 years in Loudi City

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998
The amount of waste solids (thousand tons)	20.0	20.2	24.2	27.3	30.5	32.0	42.5	50.4	56.0

The model calculation value from Equation (4) is listed in Table 2, in which the model residual and the relative residual were also given.

Fig. 1 demonstrated the difference between the model calculation value shown by line and the actual value shown by circle point.

The prominence test was carried out for the average of the residual, and t-test was employed at a prominence level $\alpha = 0.05$.

The primary hypothesis:

$$H_0: \mu = 0, H_1: \mu \neq 0$$

$\bar{q} = \frac{1}{8}\sum_{k=1}^{8} q(k) = 0.012$, in which $\bar{q}$ is the average of $q(k)$.

The average square of residual: $\sigma^2 = 0.046$, $S = \sqrt{\sigma^2} = 0.21$.

Since $T_0 = \frac{\bar{q} - \mu_0}{\sqrt{\sigma^2/n}} = \frac{\bar{q} - 0}{S/\sqrt{8}} = 0.16 < t_{0.025}(7) = 2.36$, the H_0 is accepted.

It is believed that the residual $q(k)$ approximately comply with the normal distribution $N(0, 0.21^2)$, and the probability of the residual $q(k)$ between the prediction value $\hat{x}^{(0)}(k)$ and the actual value within the range $(-0.17, 0.19)$ is 95%, which indicate obviously that the model can be well used for the prediction of the amount of waste solids.

Furthermore, the model Equation (4) can be used for the prediction the value of the amount of waste solids in Loudi City in the next 30 years from 2001—2030, which are listed in Table 3. It should be noted here that the prediction value for 1999, and 2000 are 62900 and 70600 tons, respectively, and the actual value is 64400 tons and 71000 tons respectively, the residual are not beyond the model precision range, which shows that the prediction model can well describe the variation of the amount of waste solids.

Table 2 Comparison between the amount of waste solid from calculation and the actual value

The model calculation, 10^3 tons	The actual value, 10^3 tons	The residual, $q(k)$	The relative residual, %
$\hat{x}^{(0)}(1) = 20.0$	20.0		
$\hat{x}^{(0)}(2) = 22.9$	20.2	-0.27	-13.4
$\hat{x}^{(0)}(3) = 21.3$	24.2	0.29	12.1
$\hat{x}^{(0)}(4) = 25.7$	27.3	0.16	5.9
$\hat{x}^{(0)}(5) = 30.7$	30.5	-0.02	0.7
$\hat{x}^{(0)}(6) = 35.2$	32.0	-0.32	-10.0
$\hat{x}^{(0)}(7) = 42.2$	42.5	0.03	0.7
$\hat{x}^{(0)}(8) = 48.6$	50.4	0.18	3.5
$\hat{x}^{(0)}(9) = 55.5$	56.0	0.05	0.8

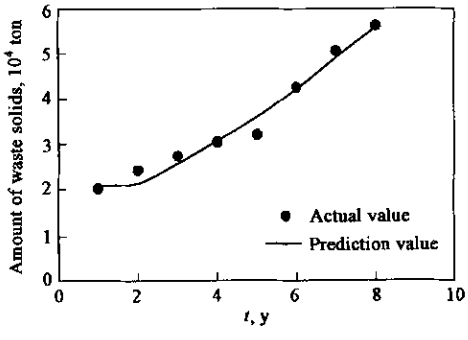

Fig. 1 Difference between the model calculation value and the actual value for the amount of waste solids

Table 3 The amount of waste solids in Loudi City in the next 30 years

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
The amount of waste solids, ten thousand tons	7.89	8.75	9.66	10.62	11.62	12.65	13.74	14.86	16.03	17.23
Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
The amount of waste solids, ten thousand tons	18.49	19.79	21.13	22.51	23.93	25.40	26.90	28.45	30.05	31.68
Year	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
The amount of waste solids, ten thousand tons	33.35	35.07	36.83	38.63	40.47	42.34	44.27	46.24	49.35	53.64

The prediction of the total amount of the waste solids in 2001—2030 is 8049000 tons. According to this prediction, the dimension of the landfill field wanted can be determined. The calculation above

transferred the non-linear differential equation into simple non-linear equation, which simplifies largely the operation. Through applying the LSM, the parameters in the model can be figured out by concise MATALAB program. The prediction precision of the model is tested by hypothesis test. Comparing with the conventional forecast method, which is mostly based on the annual increase rate, the prediction model shown a higher reliability.

3 Conclusions

Applying the gray theory model GM (1,1), the amount of urban waste solids can be well predicted by using non-linear differential equation to simulate the serials built from the second accumulation.

According to the model above, the prediction of the total amount of waste solids in Loudi City in the next 30 years is 8049000 tons.

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