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# Quantificational analysis on progress of river water quality in China

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**Abstract**

In order to understand the dynamic change of water quality in a specific period of time, a type of possibility transition matrix based on the theory of Markov process was established. The transition possibility with a weight to calculate the degree of absolute advancement was given based on the result of water quality evaluation. The concept of relative advancement was presented. It was used to evaluate the extent of water quality changed in a period of time. The method was used to calculate the degrees of relative advancement for 4 rivers in China, and the results were analyzed.

**Key words**: water quality evaluation; possibility transition matrix; Markov process; degree of advancement

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## Introduction

River pollution is a serious problem in China. In recent years, the government worked hardly to improve the water quality of rivers, and some delightful change has happened. It is necessary to know the dynamic change of water quality in a period of time. Some traditional methods can be used to evaluate the static state of river water quality on time in one place such as factor analysis models (Chen *et al*., 2002), fuzzy synthetic evaluation (Zou *et al*., 2006), matter element model (Wang *et al*., 2004), Logistic curve model (Jin *et al*., 2003), artificial neural network (Ni and Bai, 2000) and Gary analysis method (Zhang *et al*., 2004).

On the basis of the results of water quality evaluation, a quantitative value is needed to figure out how the river water quality changed in a period of time, to check whether it is improved, and to compare the change level in different rivers. A mathematical model based on Markov process is developed to quantify the change of water quality in rivers.

## 1 Mathematical model

Markov process is a special stochastic process (Guan, 1993), and has been used in forecasting (Wang *et al*., 2005; Yu *et al*., 2003; Li and Wang, 2002; Zhang *et al*., 2005; Ma and Yang, 2000; He and Xie, 1998; Zhong and Zeng, 1997; Jiang, 1996; Liu and Tan, 2002; He *et al*., 2003). Based on Markov process theory, two concepts were imported: degree of absolute progress (DAP) and degree of relative progress (DRP). DAP is used to express how much the water quality changed in a period of time, and DRP is the standardized value of DAP, which means the value of DRP varied from –1 to 1. DRP is the output of the model and the input of the model is the level of water quality at different time in different places.

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### 1.1 Establishing possibility transition matrix

According to the Environmental quality standards for surface water (GB3838-2002, State Environmental Protection Administration of China, 2002), the water quality of rivers was classified into 6 levels: level I, II, III, IV, V, under level V. In this study, we set $E$ as 6 levels, $E = \{1, 2, 3, 4, 5, 6\}$.

Take a river as an example. There are $k$ monitor places in the river and $n$ monitor samples in every place at different times, in which each one can be defined as $y_1$, $y_2$, ..., and $y_n$. Therefore, there are totally $n \times k$ monitoring samples in the river. The sum of samples that have level $i$ water quality is $M_i$. In the last monitoring, the number of samples that have level $i$ water quality is counted $m_i$. Thus, there are: $0 \leqslant m_i \leqslant k$, $0 \leqslant M_i \leqslant k$, and $\sum_i m_i = k$, $\sum_i M_i = nk$, $i \in E$.

If the level of water quality is $i$ in a monitor place at one time and the level change to $j$ level in the same place at another time, then the sum of the changes can be calculated as $n_{ij}$. Thus, there is Eq. (1).

$$0 \leqslant n_{ij} \leqslant (n-1)k \qquad \sum_{i,j} n_{ij} = (n-1)k, \quad i, j \in E \tag{1}$$

The transition possibility of which the water quality changed from $i$ level to $j$ level in this river is $n_{ij}/M_i$ according to the history data. But a situation should be considered that the water quality has no possibility to change in the last monitoring, in other words, the water quality changed, but it was not be monitored at the last monitoring. Thereby, $m_i$ should be subtracted from $M_i$. At last, the transition possibility of which the water quality changed from $i$ level to $j$ level can be calculated according to Eq. (2):

$$p_{ij} = \frac{n_{ij}}{M_i - m_i}, \quad 0 \leqslant p_{\mathrm{ij}} \leqslant 1; \quad \sum_j p_{ij} = 1 \tag{2}$$

The matrix $P = (p_{ij})_{N\times N}$ can be founded, and it is the possibility transition matrix of the river.

### 1.2 Degree of absolute progress calculation

Intending water quality can be forecasted from the possibility transition matrix of the river, but it is still unknown about how much the water quality changed in the time of monitoring. A farther work should be done, and the following situation should be considered:

(1) Endow the transition possibility with the weight, which can figure out if the water quality deteriorated or improved. If the water quality changed from low level to high level, it is deteriorated, so the weight should be negative. If the water quality changed from high level to low level, it is improved, so the weight should be positive.

(2) Endow the transition possibility with the weight, which can figure out the extent the water quality deteriorated or improved. It follows more deterioration, punishment, and improvement and encouragement.

(3) If the degrees of deterioration and improvement are the same, the absolute value of the weight is the same too.

Based on the considerations above, the equation of the weight is:

$w_{ij} = (i - j)\,|i - j|$

so $s_{ij} = (i - j)\,|i - j|\,p_{ij}, \ i, j \in E$

The matrix $S = (S_{ij})_{N\times N}$ is called advancement matrix of possibility transition matrix $P = (p_{ij})_{N\times N}$.

Therefore, the equation of DAP is:

$$\mathrm{DAP} = \sum_{i,j} s_{ij} = \sum_{i,j} (i - j)\,|i - j|\,p_{ij}, \quad i, j \in E$$

if DAP > 0, the water quality improved, if DAP < 0, the water quality deteriorated. The absolute value of DAP can figure out the degree of the water quality changed, but the value extension of DAP is unknown, therefore, it is necessary to calculate DRP.

### 1.3 Degree of relative progress calculation

After the calculation of DAP, degree of relative progress (DRP) should be calculated. When the water quality improved most, the value of DRP is 1, and when the water quality deteriorated most, the value of DRP is –1. The range of DRP is (–1, 1).

Suppose that the water quality levels of the river studied are that as shown in Table 1 when the water quality improved most. Under this situation, DAP is 25. When the water quality deteriorated most, DAP is –25. Therefore, it is defined that DRP = DAP/25, DRP ∈ (–1, 1). DRP can indicate that the degree of the water quality changes more clearly.

DRP for different rivers can be calculated and compared. If $\mathrm{DRP}_1 < \mathrm{DRP}_2 < ... < \mathrm{DRP}_n$, the water quality in the last river has the largest positive changes and that in the first river has the smallest.

**Table 1** Water quality levels of the river studied

| Month | Monitor 1 | Monitor 2 | Monitor 3 | Monitor 4 | Monitor 5 |
|---|---|---|---|---|---|
| Jan | 6 | 6 | 6 | 6 | 6 |
| Feb | 1 | 1 | 1 | 1 | 1 |
| Mar | 1 | 1 | 1 | 1 | 1 |
| Apr | 1 | 1 | 1 | 1 | 1 |
| May | 1 | 1 | 1 | 1 | 1 |
| Jun | 1 | 1 | 1 | 1 | 1 |

## 2 Results and discussion

The study was applied to develop a model to analyze the water quality changes of four rivers in China in 2005. These four rivers are the Songhua River, the Haihe River, the Yellow River, and the Yangtze River. As mentioned above, the input of the model is the water quality level of each river at different time and different monitoring places. The source data for study were provided by Environmental Monitoring Center of China (CNEMC). There are 27 monitoring points along the four rivers. The level of water quality was evaluated every week. Taking the Songhua River as an example, the level of water quality is given in Table 2.

**Table 2** Water quality level of the Songhua River in 2005

| Month | Monitor 1 | Monitor 2 | Monitor 3 | Monitor 4 | Monitor 5 |
|---|---|---|---|---|---|
| Jan | 3 | 4 | 3 | 6 | 2 |
| Feb | 3 | 4 | 2 | 4 | 3 |
| Mar | 3 | 4 | 2 | 4 | 4 |
| Apr | 3 | 4 | 2 | 4 | 3 |
| May | 3 | 3 | 2 | 4 | 5 |
| Jun | 4 | 3 | 2 | 3 | 5 |
| Jul | 4 | 2 | 2 | 4 | 4 |
| Aug | 4 | 3 | 2 | 3 | 4 |
| Sep | 5 | 3 | 2 | 4 | 3 |
| Oct | 4 | 2 | 2 | 6 | 4 |
| Nov | 4 | 3 | 2 | 5 | 3 |
| Dec | 4 | 3 | 2 | 5 | 2 |

The results of $m_i$, $M_i$, and $n_{ij}$ are calculated as follows:

$$\begin{array}{llllll}
m_1 = 0 & m_2 = 2 & m_3 = 1 & m_4 = 1 & m_5 = 1 & m_6 = 0 \\
M_1 = 0 & M_2 = 15 & M_3 = 18 & M_4 = 20 & M_5 = 5 & M_6 = 2 \\
n_{11} = 0 & n_{12} = 0 & n_{13} = 0 & n_{14} = 0 & n_{15} = o & n_{16} = 0 \\
n_{21} = 0 & n_{22} = 10 & n_{23} = 3 & n_{24} = 0 & n_{25} = 0 & n_{26} = 0 \\
n_{31} = 0 & n_{32} = 4 & n_{33} = 7 & n_{34} = 5 & n_{35} = 1 & n_{36} = 0 \\
n_{41} = 0 & n_{42} = 0 & n_{43} = 6 & n_{44} = 11 & n_{45} = 1 & n_{46} = 1 \\
n_{51} = 0 & n_{52} = 0 & n_{53} = 0 & n_{54} = 2 & n_{55} = 2 & n_{56} = 0 \\
n_{61} = 0 & n_{62} = 0 & n_{63} = 0 & n_{64} = 1 & n_{65} = 1 & n_{66} = 0
\end{array} \tag{3}$$

The possibility transition matrix is then established based on the results.

$$P_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 10/13 & 3/13 & 0 & 0 & 0 \\
0 & 4/7 & 7/17 & 5/17 & 1/17 & 0 \\
0 & 0 & 6/19 & 11/19 & 1/19 & 1/19 \\
0 & 0 & 0 & 1/2 & 1/2 & 0 \\
0 & 0 & 0 & 1/2 & 1/2 & 0
\end{bmatrix} =$$

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.769 & 0.231 & 0 & 0 & 0 \\
0 & 0.235 & 0.412 & 0.294 & 0.059 & 0 \\
0 & 0 & 0.316 & 0.579 & 0.053 & 0.053 \\
0 & 0 & 0 & 0.5 & 0.5 & 0 \\
0 & 0 & 0 & 0.5 & 0.5 & 0
\end{bmatrix} \tag{4}$$

**Table 3** Water quality level of the Haihe River in 2005

| Month | Monitor 1 | Monitor 2 | Monitor 3 | Monitor 4 | Monitor 5 | Monitor 6 | Monitor 7 | Monitor 8 |
|---|---|---|---|---|---|---|---|---|
| Apr | 2 | 2 | 3 | 1 | 6 | 6 | 1 | 6 |
| May | 2 | 2 | 3 | 2 | 6 | 6 | 1 | 5 |
| Jun | 2 | 2 | 4 | 2 | 6 | 6 | 1 | 6 |
| Jul | 2 | 3 | 6 | 2 | 6 | 6 | 1 | 6 |
| Aug | 3 | 6 | 5 | 2 | 5 | 6 | 1 | 6 |
| Sep | 3 | 4 | 5 | 2 | 6 | 6 | 1 | 6 |
| Oct | 2 | 3 | 4 | 2 | 6 | 6 | 1 | 6 |
| Nov | 2 | 4 | 4 | 2 | 3 | 6 | 1 | 6 |
| Dec | 2 | 4 | 4 | 2 | 6 | 6 | 1 | 6 |

Therefore, the advancement matrix $S_1$ can be calculated as Eq. (5), based on the equation of DAP, $DAP_1 = 2.525$, $DRP_1 = DAP_1/25 = 0.101$.

$$S_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.231 & 0 & 0 & 0 \\ 0 & 0.235 & 0 & -0.294 & -0.236 & 0 \\ 0 & 0 & 0.316 & 0 & -0.053 & -0.212 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0.5 & 0 \end{bmatrix} \quad (5)$$

The calculation of DRP for other three rivers is the same. The DRP of the Haihe River, the Yellow River and the Yangtze River are: $DRP_2 = -0.136$, $DRP_3 = 0.056$, $DRP_4 = 0.147$, respectively. The DPR value for the four rivers were in the sequence of $DRP_2 < DRP_3 < DRP_1 < DRP_4$.

The calculation result showed that DRP of the Haihe River is negative, it can be concluded that the water quality deteriorated. The same conclusion can be directly obtained from Table 3, because the water quality of the Haihe River in monitor 2, monitor 3 and monitor 4 deteriorated gradually. The level of water quality in monitor 3 and monitor 4 became worse than class V at one time, and the other positions remained the same. Generally speaking, the water quality of the Haihe River was getting worse. This is consistent with the calculation results.

The water quality of the other three rivers turned to be better in 2005, such as the Songhua River. As shown in Table 2, the water quality in monitor 2, monitor 3 and monitor 4 improved at a certain extent. The water quality in monitor 1 deteriorated slightly, and in monitor 5 it fluctuated randomly. They all affected the results of the calculation. Although the DRP is positive, the absolute value is small. This indicates that although the water quality of the Songhua River improved in 2005, the improvement is not significant. The situation of the Yellow River and the Yangtze River is similar; the water quality did not improve much. The water quality in the Yangtze River has the most improvement among the four rivers, while the Yellow River has the least.

## 3 Conclusions

The study established a mathematical model based on Markov process to analyze the changes of water quality in a period of time. The key point is the introduction of the concept DRP. The developed model has been applied to four Chinese rivers in 2005. The calculation results showed that the value of DRP can reflect the actual situation of the river water quality changes. The conclusion of the case studies indicated that the water quality improvements of four rivers are limited, and the Haihe River became worse. More attention should be paid to protecting the rivers from being polluted.

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